Part VI

第六部分

String Theories

弦理论

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A Lightning Introduction to String Theory

弦理论简引

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摘要

We give a lightning introduction to critical string theory, including the 26-dimensional bosonic string, the 10-dimensional superstrings and heterotic strings with and without spacetime supersymmetry. We also discuss open strings and D-branes, as well as the orientifold constructions, in ten dimensions.

我们对临界弦理论作了一个快速介绍，涵盖 26 维玻色弦、10 维超弦，以及具有和不具有时空超对称的杂化弦。我们还讨论了十维中的开弦、D 膜，以及定向模构造。

Keywords

关键词

String theory - Bosonic string - Superstrings - Heterotic strings - D-branes - Orientifolds

弦理论 - 玻色弦 - 超弦 - 杂化弦 - D 膜 - 定向模

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Motivating String Theory

弦理论导论

String theory is currently regarded as a most promising framework in which quantum gravity is unified with gauge interactions and matter. Its origins can be traced back to the seminal work of Veneziano [1], who proposed the simple expression

弦论目前被认为是将量子引力与规范相互作用和物质统一起来的最具前景的理论框架。它的起源可以追溯到韦内齐亚诺的开创性工作 [1]，他提出了这个简洁表达式

$$A(s,t,u) = B(-\alpha(s),-\alpha(t)) + B(-\alpha(s),-\alpha(u)) + B(-\alpha(t),-\alpha(u)), \quad (1)$$

for a relativistic $2 \to 2$ amplitude obeying the requirements of Regge trajectories and high-energy behaviour along with crossing symmetry, in order to account for the large amount of experimental data on hadronic resonances produced throughout the 1960s. $B(-\alpha(s),-\alpha(t))$ is the Euler beta function with $\alpha(z) = \alpha' z + \alpha_0$, clearly symmetric under the exchange of its arguments, which guarantees the full $s \leftrightarrow t \leftrightarrow u$ crossing symmetry in (1), where s,t,u are the familiar Mandelstam variables. Using standard properties of the Euler gamma function, it follows that $B(-\alpha(s),-\alpha(t))$ has an infinite number of equally spaced poles at $t = (n-\alpha_0)/\alpha'$, $n = 0,1,2,\ldots$, with residues that are order- n polynomials in s . This incorporates the linearly rising Regge trajectories

适用于满足雷杰轨迹、高能行为以及交叉对称性要求的相对论 $2 \to 2$ 振幅，用以解释 20 世纪 60 年代积累的大量强子共振实验数据。$B(-\alpha(s),-\alpha(t))$ 是满足 $\alpha(z) = \alpha' z + \alpha_0$ 的欧拉 β 函数，在交换其自变量时具有明显对称性，这保证了 (1) 式中完整的 $s \leftrightarrow t \leftrightarrow u$ 交叉对称性，式中 s,t,u 是我们熟知的曼德尔施塔姆变量。利用欧拉伽马函数的标准性质可推得，$B(-\alpha(s),-\alpha(t))$ 在 $t = (n-\alpha_0)/\alpha'$ 、$n = 0,1,2,\ldots$ 处存在无穷多个等间距极点，其留数是 s 上的 n 阶多项式。这就纳入了线性上升的雷杰轨迹

$$m^2 = (J-\alpha_0)/\alpha', \quad (2)$$

observed in many hadronic resonances, where $J = n$ is the maximal spin of the particle exchanged in the t channel, and a similar behaviour is clearly present in the s and u channels. Experimental fits required the Regge slope α' to be of order $\alpha' \sim 1\mathrm{GeV}^{-2}$, while the Regge intercept α_0 was left undetermined. Subsequent works dealing with unitarity and the absence of ghosts, led to no-ghost theorems fixing $\alpha_0 = 1$ and the dimension of spacetime to $D = 26$.

> 已在许多强子共振中观测到，其中 $J = n$ 是 t 道中交换粒子的最大自旋，s 道和 u 道也明确存在类似行为。实验拟合要求雷杰斜率 α' 量级为 $\alpha' \sim 1\mathrm{GeV}^{-2}$ ，而雷杰截距 α_0 并未确定。后续关于幺正性和无鬼条件的研究得出了无鬼定理，将 $\alpha_0 = 1$ 和时空维数固定为 $D = 26$ 。

In the high energy regime, the Stirling formula implies the asymptotic behaviour $\sim s^{\alpha(t)}$ for the beta function, valid in the Regge limit of large s and fixed t . For sufficiently negative t , the high energy behaviour of the Veneziano amplitude is thus extremely soft, something which can only occur if an infinite number of particles are exchanged in the t channel. Similarly, the Veneziano amplitude at high energy and fixed angle is exponentially suppressed $\sim f(\theta)^{-\alpha(s)}$, with f a given function of the scattering angle θ .

> 在高能区域，斯特林公式给出了 β 函数的渐近行为 $\sim s^{\alpha(t)}$ ，该行为在大 s 、固定 t 的雷吉极限下成立。当 t 足够负时，韦内齐亚诺振幅的高能行为会极弱，这种情况只有当 t 道中交换无穷多粒子时才会发生。同理，高能固定角下的韦内齐亚诺振幅会按指数压低 $\sim f(\theta)^{-\alpha(s)}$ ，其中 f 是散射角 θ 的给定函数。

Soon after the work of the Veneziano, it was realised [2-5] that the linear Regge trajectory could be reproduced by the mechanical model of a vibrating quantum string. In this framework, the interactions are no longer localised in spacetime, which explains the softness of the high energy limit.

> 威尼齐亚诺的工作问世后不久，研究人员就发现 [2-5]，线性雷杰轨迹可以通过振动量子弦的力学模型重现。在该框架中，相互作用不再定域于时空内，这也解释了高能极限的软性质。

This extreme UV softness of the dual models emerging from (1), together with their prediction of a massless spin-2 (hadronic) particle, and the surprising requirement for the dimensionality of spacetime, appeared to be incurable problems. The discovery of quantum chromodynamics (QCD) as the correct theory of strong interactions, eventually led to their demise.

> 由式 (1) 得出的对偶模型在极紫外区的软特性，加上它预测存在无质量自旋 2(强子) 粒子，还有对时空维度出人意料的要求，这些似乎都是无法解决的问题。量子色动力学 (QCD) 被发现是描述强相互作用的正确理论，最终导致这些对偶模型被淘汰。

Actually, the presence of a massless spin-2 particle, and the soft behaviour of the dual models turned out to be a blessing in disguise when, thanks to the seminal work of Yoneya [6,7] and Scherk and Schwarz [8,9], it was proposed that the models of relativistic strings should rather be interpreted as a theory of quantum gravity, with α' now being related to the Planck scale. In this description, the apparently ”wrong” dimensionality of spacetime is no longer a problem, since spacetime becomes dynamical and can undergo a spontaneous compactification down to four dimensions.

事实上，在米山哲也 [6,7] 以及谢尔克与施瓦茨 [8,9] 的开创性工作提出，相对论弦模型应当被诠释为量子引力理论，其中 α' 与普朗克尺度相关联之后，无质量自旋 2 粒子的存在以及对偶模型的软行为才被发现是因祸得福。在这一描述中，看似“错误”的时空维度不再是问题，因为时空本身是动力学的，可通过自发紧致化降至四维。

Ever since, string theory underwent a series of momentous developments which contributed to sharpening our understanding of the subject and led to the many diverse steps which eventually shaped it into its present form. The string section of the handbook aims at giving an introduction to this exciting field, ranging from formal aspects to phenomenological implications in particle physics and cosmology.

从那以后，弦理论经历了一系列重大发展，加深了我们对这一领域的理解，促成了诸多不同方向的进展，最终将其塑造为如今的形态。本手册的弦论部分旨在介绍这一令人振奋的领域，内容涵盖从形式层面到其在粒子物理与宇宙学中的唯象学意义。

This chapter is organised as follows. In section ”The Bosonic String” we introduce the bosonic string, its quantisation and discuss its light spectrum in some detail. The partition function for closed bosonic strings is derived using the operatorial approach and its interpretation in terms of the torus amplitude is given. In section ”Two-Dimensional Conformal Field Theory” we focus on the salient properties of the conformal field theory living on the two-dimensional string world-sheet, including the Virasoro algebra and the b, c ghost system. In section ”String Perturbation Theory,” we discuss the path integral and BRST quantisation, the vertex operators for physical fields are derived and the structure of string perturbation theory is presented. Conformal invariance on a general background and the associated low-energy effective action are briefly discussed. The Shapiro-Virasoro and Veneziano amplitudes are derived in some detail, along with a path integral evaluation of the torus vacuum amplitude. Fermionic strings and their light-cone quantisation are presented in section ”Fermionic Strings,” with emphasis on modular invariance and the GSO projections, leading to the construction of the ten-dimensional type IIA, IIB, 0A, 0B theories. We briefly comment on the concept of orbifolds, within the context of constructing 0A, 0B from type IIA, IIB superstrings. Heterotic strings are introduced in section ”Heterotic Strings,” along with their ten-dimensional vacuum configurations with and without space-time supersymmetry. Section ”Open Strings and D-Branes” contains a discussion of open strings and their boundary conditions, leading to the notion of D-branes. Their tensions and R-R charges are computed by studying the corresponding cylinder (transverse channel) amplitudes. Finally, the orientifold construction, involving orientifold planes and D-branes, is presented in section ”Orientifolds.” The ten-dimensional Type I superstring and the Sugimoto vacuum, together with the various orientifolds of type 0B superstrings are also discussed.

本章结构安排如下: 在“玻色弦”一节中，我们介绍玻色弦及其量子化，并详细讨论其轻谱。我们采用算符方法推导闭玻色弦的配分函数，给出其在环面振幅框架下的诠释。在“二维共形场论”一节中，我们聚焦于二维弦世界面上共形场论的核心性质，包括 Virasoro 代数与 b, c 鬼系统。在“弦微扰论”一节中，我们讨论路径积分与 BRST 量子化，推导物理场的顶点算符，介绍弦微扰论的结构，简要讨论一般背景下的共形不变性及对应的低能有效作用量。我们详细推导了 Shapiro-Virasoro 振幅与 Veneziano 振幅，同时对环面真空振幅做了路径积分计算。在“费米弦”一节中，我们介绍费米弦及其光锥量子化，重点讨论模不变性与 GSO 投影，以此构造十维 IIA、IIB、0A、0B 型理论。我们还在从 IIA、IIB 超弦构造 0A、0B 理论的背景下，简要评述了轨形的概念。在“杂化弦”一节中，我们介绍杂化弦，以及存在和不存在时空超对称的十维真空构型。“开弦与 D 膜”一节讨论开弦及其边界条件，由此引出 D 膜的概念。我们通过研究对应的柱面 (横道) 振幅计算了 D 膜的张力与 R-R 荷。最后，在“定向模”一节中介绍包含定向模平面与 D 膜的定向模构造，同时讨论十维 I 型超弦、杉本真空，以及 0B 型超弦的各类定向模。

The topics outlined in this review clearly do not exhaust this very rich field, and the style of the presentation reflects the authors' idiosyncrasies and biases. Given the lightning spirit of the exposition, the list of provided references is clearly incomplete and we apologise in advance for the inevitable omissions. We warmly encourage the interested reader to consult the many excellent books [10-27] and reviews [28-45] available in the literature, many of which provide detailed references to the original papers.

本综述梳理的主题显然未能涵盖这个内容极为丰富的领域, 表述风格也反映了作者本人的研究特点与偏好。由于本文阐述力求简洁明快，所列参考文献必然不够完整，我们对无法避免的遗漏提前致歉。我们诚挚建议有兴趣的读者查阅文献中诸多优秀的专著 [10-27] 与综述 [28-45]，其中多数都给出了原始论文的详细引用。

We also refer to the various other contributions to this handbook for an introduction to the tentacular developments and applications of string theory.

关于弦理论各类广阔的发展与应用，我们也建议读者参考本手册中的其他相关综述。

The Bosonic String

玻色弦

Consider a point-particle of mass m freely moving in D -dimensional Minkowski spacetime of signature $(-,+,\dots,+)$. It traces a world-line $x^\mu(\tau)$ parametrised by the proper time τ , whose invariant length $ds^2 = -\eta_{\mu\nu}dx^\mu dx^\nu$ is defined by the Minkowski metric $\eta_{\mu\nu}$, where $\mu, \upsilon = 0, \dots, D-1$. The dynamics is controlled by the action

考虑一个质量为 m 的点粒子，在号差为 $(-,+,\dots,+)$ 的 D 维闵氏时空中自由运动。它扫出一条由固有时 τ 参数化的世界线 $x^\mu(\tau)$ ，其不变长度 $ds^2 = -\eta_{\mu\nu}dx^\mu dx^\nu$ 由闵氏度规 $\eta_{\mu\nu}$ 定义，其中 $\mu, \upsilon = 0, \dots, D-1$ 。动力学由如下作用量描述

$$S = -m\int ds = -m\int d\tau\sqrt{-\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}, \tag{3}$$

proportional to the invariant length of its world-line. For higher-dimensional objects, the natural generalisation is in terms of an action proportional to the area of the hypersurface traced by the object as it moves in spacetime. In the case of a string of tension $T = 1/2\pi\alpha'$, the action reads

该作用量正比于世界线的不变长度。对于高维物体，自然的推广就是让作用量正比于物体在时空中运动时扫出的超曲面面积。对于弦张力为 $T = 1/2\pi\alpha'$ 的弦，作用量形式为

$$S = -T \int_{\sum} d\tau d\sigma \sqrt{-\det\left(\eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu\right)}, \tag{4}$$

as proposed by Nambu and Goto. Here, τ and σ denote the proper time and the proper length on the worldsheet $\sum$, which is embedded into spacetime by the maps $X^\mu(\tau,\sigma)$. Furthermore $\partial_a, a = 0,1$ denotes the partial derivatives with respect to τ and $\sigma \in [0,1]$, respectively. One of the main differences with the point-particle case is the fact that strings come in two different topologies: they may be either open or closed and, thus, require suitable periodicity or boundary conditions at $\sigma = 0,1$.

这是南布和吉塔提出的。此处 τ 和 σ 分别表示世界面 $\sum$ 上的固有时和固有长度，世界面通过映射 $X^\mu(\tau,\sigma)$ 嵌入时空；此外 $\partial_a, a = 0,1$ 分别表示对 τ 和 $\sigma \in [0,1]$ 的偏导数。弦和点粒子的一个主要区别是弦有两种不同拓扑: 开弦或闭弦，因此需要在 $\sigma = 0,1$ 处施加合适的周期性条件或边界条件。

Notice that the actions (3) and (4) are non-polynomial, which poses difficulties for quantising the theories. The situation may be remedied by introducing new nondynamical auxiliary fields, which act as Lagrange multipliers and remove the square roots. For the point particle, the new action reads

注意作用量 (3) 和 (4) 都是非多项式的，这给理论量子化带来了困难。我们可以引入新的非动力学辅助场作为拉格朗日乘子消去根号，解决这个问题。对于点粒子，新作用量形式为

$$S = \frac{1}{2}\int d\tau \left(e^{-1}\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu - em^2\right). \tag{5}$$

The equation of motion (e.o.m.) for the auxiliary field e acts as the constraint $e^{-2}\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + m^2 = 0$, which one may solve for e and plug back into the action to recover (3). The constraint actually imposes the mass-shell condition $p^2 + m^2 = 0$, which corresponds to the vanishing of the energy-momentum tensor of the one-dimensional theory living on the world-line of the point-particle. This is so because the $e(\tau)$ is naturally identified as the einbein on the world-line, making reparametrisation invariance manifest.

辅助场 e 的运动方程给出约束 $e^{-2}\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + m^2 = 0$ ，我们可以对 e 求解，再代回作用量就能得到 (3) 式。这个约束实际上给出了质壳条件 $p^2 + m^2 = 0$ ，对应点粒子世界线上一维理论的能量动量张量为零。这是因为 $e(\tau)$ 可以自然对应世界线上的爱因贝因，让重参数化不变性变得明显。

Following a similar route in the case of the string, one introduces the world-sheet metric g_{ab} of signature $(-,+)$, which plays the role of the Lagrange multipliers, and the action

对弦采用类似的方法，我们引入号差为 $(-,+)$ 的世界面度规 g_{ab} ，它起到拉格朗日乘子的作用，对应的作用量为

$$S=-\frac{T}{2}\int_{\Sigma} d^2\sigma\sqrt{-g}g^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}, \tag{6}$$

as proposed by Brink, di Vecchia, Howe [46] and Deser, Zumino [47], and known as the Polyakov action. Also in this case, the e.o.m. for g_{ab} imposes the vanishing of the two-dimensional energy-momentum tensor

> 这是由 Brink、di Vecchia、Howe[46] 以及 Deser、Zumino[47] 提出的，称为波利亚科夫作用量。同样在这个情况下，g_{ab} 的运动方程要求二维能量动量张量为零

$$T_{ab}=-\frac{2}{T}\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{ab}}=\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}-\frac{1}{2}g_{ab}g^{cd}\partial_c X^\mu \partial_d X^\nu \eta_{\mu\nu}=0. \tag{7}$$

Again, solving the constraint and plugging it back into (6) reproduces (4). Whether or not this equivalence persists at the quantum level is an open problem. In the following, we shall take the Polyakov action as the starting point for the quantisation of strings.

> 求解约束后代入 (6) 式就可以重新得到 (4) 式。这种等价性在量子层面是否仍然成立目前仍是开放问题。在下文中，我们将以波利亚科夫作用量作为弦量子化的起点。

Aside from D -dimensional Poincaré symmetry and two-dimensional diffeomor-phisms $\sigma^a \to \sigma'^a(\sigma)$, the Polyakov action has the remarkable property of being invariant under Weyl rescaling of the two-dimensional metric, $g_{ab} \to e^{\omega(\sigma)}g_{ab}$, which is reflected in the vanishing of the trace of the energy-momentum tensor, $g^{ab}T_{ab} = 0$. This is a special property of two-dimensional world-sheets which plays a crucial role in the quantisation of the theory and singles out strings from higher-dimensional objects.

> 除了 D 维庞加莱对称性和二维微分同胚 $\sigma^a \to \sigma'^a(\sigma)$ ，波利亚科夫作用量还有一个出色的性质: 它在二维度规的外尔缩放 $g_{ab} \to e^{\omega(\sigma)}g_{ab}$ 下不变，这体现为能量动量张量的迹为零，即 $g^{ab}T_{ab}=0$ 。这是二维世界面独有的特殊性质，在理论量子化中发挥关键作用，也是弦区别于更高维延展物体的特点。

We can employ two-dimensional reparametrisation invariance and Weyl rescaling to locally gauge fix the world-sheet metric to $g_{ab}=\eta_{ab}$, usually called the conformal gauge. With this choice, the e.o.m. for X^μ becomes simply the two-dimensional d’Alembert equation, $\partial_+\partial_- X=0$, whose general solution $X^\mu(\tau,\sigma) = X_L^\mu(\sigma^+)+X_R^\mu(\sigma^-)$ is the superposition of left-moving waves X_L and right-moving waves X_R , with $\sigma^\pm=\tau\pm\sigma$. The tracelessness of the energy-momentum tensor implies that $T_{\pm\mp}=0$, while

> 我们可以利用二维重参数化不变性与外尔标度变换，将世界面度规局部规范固定为 $g_{ab}=\eta_{ab}$ ，这通常称为共形规范。在此选择下，X^μ 的运动方程可简化为二维达朗贝尔方程 $\partial_+\partial_- X=0$ ，其通解 $X^\mu(\tau,\sigma)= X_L^\mu(\sigma^+)+X_R^\mu(\sigma^-)$ 是左行波 X_L 与右行波 X_R 的叠加，满足 $\sigma^\pm=\tau\pm\sigma$ 。能量动量张量的无迹性给出 $T_{\pm\mp}=0$ ，而

$$T_{\pm\pm}=\partial_\pm X^\mu \partial_\pm X^\upsilon \eta_{\mu\upsilon}. \tag{8}$$

Its conservation then becomes $\partial_\mp T_{\pm\pm}=0$, so that each component is separately conserved.

> 其守恒律可表示为 $\partial_\mp T_{\pm\pm}=0$ ，因此每个分量分别守恒。

In order to extract the e.o.m. from (6), one needs to specify appropriate periodicity or boundary conditions. Closed strings clearly satisfy $X^\mu(\tau,\sigma+1) = X^\mu(\tau,\sigma)$ so that we can Fourier expand the solution as

为了从式 (6) 推导运动方程，需要指定合适的周期性条件或边界条件。闭弦显然满足 $X^\mu(\tau,\sigma+1) = X^\mu(\tau,\sigma)$，因此我们可以对解做傅里叶展开得到:

$$X_L^\mu(\sigma^+) = \frac{1}{2}x_0^\mu + \pi\alpha' p^\mu \sigma^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-2\pi i n\sigma^+},$$

(9)

$$X_R^\mu(\sigma^-) = \frac{1}{2}x_0^\mu + \pi\alpha' p^\mu \sigma^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n^\mu}{n}e^{-2\pi i n\sigma^-}.$$

It is straightforward to see that x_0^μ corresponds to the centre of mass position of the string, which freely moves with momentum p^μ. Furthermore, the reality of X^μ implies the relations $\alpha_{-n}^\mu = \left(\alpha_n^\mu\right)^*$, and similarly for the right-moving coefficients.

不难看出 x_0^μ 对应弦的质心位置，它随动量 p^μ 自由运动。此外，X^μ 的实性给出关系 $\alpha_{-n}^\mu = \left(\alpha_n^\mu\right)^*$，右行系数也满足类似关系。

In the case of open strings, the boundary term arising from the Polyakov action is $\delta X^\mu \partial_\sigma X_\mu\big|_{\sigma=0}^{\sigma=1} = 0$, and one has to distinguish among three inequivalent cases: Neumann (N) boundary conditions $\partial_\sigma X^\mu = 0$ at both endpoints, Dirichlet (D) boundary conditions $\partial_\tau X^\mu = 0$ at both endpoints, or N boundary condition at one endpoint and D on the other. In all cases, the left-moving waves are reflected into the right-moving ones so that X_L^μ and X_R^μ are no longer independent, and

对于开弦，波利亚科夫作用量给出的边界项为 $\delta X^\mu \partial_\sigma X_\mu\big|_{\sigma=0}^{\sigma=1} = 0$，需要区分三种不等价情况: 两个端点都采用诺依曼 (N) 边界条件 $\partial_\sigma X^\mu = 0$，两个端点都采用狄利克雷 (D) 边界条件 $\partial_\tau X^\mu = 0$，或是一个端点采用 N 边界条件、另一个端点采用 D 边界条件。在所有情况下，左行波都会反射为右行波，因此 X_L^μ 和 X_R^μ 不再独立，且

$$X^\mu(\sigma,\tau) = x_0^\mu + 2\pi\alpha' p^\mu\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-i\pi n\tau}\cos(n\pi\sigma), \tag{10}$$

for NN boundary conditions. In the DD case, the solution reads

对应 NN 边界条件。对于 DD 情形，解为

$$X^\mu(\sigma,\tau) = x_0^\mu + \delta^\mu\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-i\pi n\tau}\sin(n\pi\sigma), \tag{11}$$

which clearly shows that the centre of mass is no longer moving, while x_0^μ and $x_0^\mu + \delta^\mu$ are the fixed positions of the two endpoints $\sigma = 0, 1$. In ND case there are no zero modes, while the frequencies are half-integral.

可以清楚看出，质心不再运动，x_0^μ 和 $x_0^\mu + \delta^\mu$ 是两个端点 $\sigma = 0, 1$ 的固定位置。对于 ND 情形，不存在零模，频率为半整数。

In canonical quantisation, one imposes the equal-time commutator relation $[X^\mu(\tau,\sigma), \Pi^\upsilon(\tau,\sigma')] = i\eta^{\mu\upsilon}\delta(\sigma-\sigma')$ between the coordinates X^μ and their conjugate momenta $\Pi^\upsilon = \partial_\tau X^\upsilon / 2\pi\alpha'$. In terms of the Fourier coefficients, one finds

在正则量子化中，我们要求坐标 X^μ 和其共轭动量 $\Pi^\upsilon = \partial_\tau X^\upsilon / 2\pi\alpha'$ 满足等时对易关系 $[X^\mu(\tau,\sigma), \Pi^\upsilon(\tau,\sigma')] = i\eta^{\mu\upsilon}\delta(\sigma-\sigma')$ 。用傅里叶系数表示可以得到:

$$\left[\alpha_m^\mu, \alpha_n^\upsilon\right] = m\delta_{m+n}\eta^{\mu\upsilon},\ \left[\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\upsilon\right] = m\delta_{m+n}\eta^{\mu\upsilon},\ \left[\alpha_m^\mu, \tilde{\alpha}_n^\upsilon\right] = 0, \tag{12}$$

that, together with the Hermiticity conditions $\alpha_{-n}^\mu = \left(\alpha_n^\mu\right)^\dagger$, identifies them as creation and annihilation operators for an infinite number of harmonic oscillators. The naive construction of the Fock space is plagued by negative-norm states, akin to the canonical quantisation of Maxwell theory. Indeed, $\left\|\alpha_{-n}^\mu|0\rangle\right\|^2 = n\eta^{\mu\mu}$ is negative in the temporal direction. As in the Gupta-Bleuler procedure, these ghostlike states are eliminated once the vanishing of the energy-momentum tensor is weakly imposed on the physical spectrum. Actually, in string theory the construction of a ghost-free spectrum is more subtle since T_{ab} has a quadratic dependence on the oscillators. A careful though tedious analysis reveals that it is only possible in $D = 26$ dimensions.

结合厄米性条件 $\alpha_{-n}^\mu = \left(\alpha_n^\mu\right)^\dagger$ ，这可以将它们确定为无穷多个谐振子的产生和湮灭算符。与麦克斯韦理论的正则量子化类似，福克空间的朴素构造会被负范数态困扰。事实上，$\left\|\alpha_{-n}^\mu|0\rangle\right\|^2 = n\eta^{\mu\mu}$ 在时间方向上为负。如同在古普塔-布勒勒 procedure 中一样，当我们对物理谱弱施加能量动量张量为零的条件后，类鬼态就会被消除。实际上，弦论中无鬼谱的构造更加微妙，因为 T_{ab} 对谐振子存在二次依赖。细致却繁琐的分析表明，这仅在 $D = 26$ 维下可行。

An alternative way of quantising the theory which avoids negative-norm states is light-cone quantisation [48], that we shall now follow. The conservation of T_{ab} actually implies the existence of an infinite number of Noether charges, since $\partial_\mp\left(f_\pm\left(\sigma^\pm\right)T_{\pm\pm}\right) = 0$ for arbitrary functions $f_\pm$. This suggests that the Polyakov action in the conformal gauge enjoys infinite residual symmetries. In fact, a combined action of Weyl rescaling $\delta g_{ab} = \omega g_{ab}$ and diffeomorphisms $\delta\sigma^\pm = \xi^\pm$ can leave the two-dimensional Minkowski metric invariant provided

一种可以避免负范数态的量子化替代方法是光锥量子化 [48]，我们接下来将采用这种方法。T_{ab} 的守恒实际上意味着存在无穷多诺特荷，因为对任意函数 $f_\pm$ 都有 $\partial_\mp\left(f_\pm\left(\sigma^\pm\right)T_{\pm\pm}\right) = 0$ 。这表明共形规范下的 Polyakov 作用量拥有无穷多剩余对称性。事实上，只要满足条件，外尔缩放 $\delta g_{ab} = \omega g_{ab}$ 和微分同胚 $\delta\sigma^\pm = \xi^\pm$ 的联合作用就可以保持二维闵氏度规不变

$$\partial_\mp\xi^\pm = 0 \ \text{ and } \ \omega = -\partial_+\xi^+ - \partial_-\xi^-. \tag{13}$$

The finite transformations are then given by the arbitrary chiral reparametrisations $\sigma^\pm \to \sigma'^\pm\left(\sigma^\pm\right)$, which implies $\tau' = \sigma'^+\left(\sigma^+\right) + \sigma'^-\left(\sigma^-\right)$ also satisfies the d’Alembert equation. The light-cone quantisation identifies the proper time τ with the $X^+ = \left(X^0 + X^{D-1}\right)/\sqrt{2}$ coordinate

有限变换由任意 chiral 重参数化 $\sigma^{\pm} \to \sigma'^{\pm}(\sigma^{\pm})$ 给出，这意味着 $\tau' = \sigma'^{+}(\sigma^{+}) + \sigma'^{-}(\sigma^{-})$ 同样满足达朗贝尔方程。光锥量子化将固有时 τ 等同于 $X^{+} = \left(X^{0} + X^{D-1}\right)/\sqrt{2}$ 坐标

$$X^{+} = x^{+} + 2\pi\alpha' p^{+}\tau \tag{14}$$

thus eliminating the oscillators along this direction. This has the advantage of linearising the constraint (7), which can be solved for $X^{-} = \left(X^{0} - X^{D-1}\right)/\sqrt{2}$,

从而消去了这个方向上的谐振子。这种方法的优势是将约束 (7) 线性化，之后就可以对 $X^{-} = \left(X^{0} - X^{D-1}\right)/\sqrt{2}$ 求解,

$$\partial_{\pm}X^{-} = \frac{1}{2\pi\alpha' p^{+}}\partial_{\pm}X^{i}\partial_{\pm}X^{i}, \tag{15}$$

with $i = 1, \ldots, D-2$ labelling the transverse coordinates. In terms of oscillators,

其中 $i = 1, \ldots, D-2$ 标记横向坐标。用谐振子表示为

$$\alpha_{n}^{-} = \frac{1}{2\sqrt{2\alpha'}p^{+}}\sum_{m\in\mathbb{Z}}\alpha_{n-m}^{i}\alpha_{m}^{i}, \tag{16}$$

for open strings with NN boundary conditions, while

对应具有 NN 边界条件的开弦，而

$$\alpha_{n}^{-} = \frac{1}{\sqrt{2\alpha'}p^{+}}\sum_{m\in\mathbb{Z}}\alpha_{n-m}^{i}\alpha_{m}^{i}, \tag{17}$$

for closed strings, together with a similar equation for the right-movers. Here, we have introduced $\alpha_0^{\mu} = \sqrt{2\alpha'}p^{\mu}$ for open strings, while $\alpha_0^{\mu} = \tilde{\alpha}_0^{\mu} = \sqrt{\alpha'/2}p^{\mu}$. From Eqs. (16) and (17), it is clear that the physical oscillators are only those in the transverse directions and, hence, automatically build a Fock space of positive norm. Among all modes, the $n = 0$ one is special since it provides the mass-shell condition

对应闭弦,右动模式也满足类似的方程。这里我们对开弦引入了 $\alpha_0^{\mu} = \sqrt{2\alpha'}p^{\mu}$,而 $\alpha_0^{\mu} = \tilde{\alpha}_0^{\mu} = \sqrt{\alpha'/2}p^{\mu}$ 。从式 (16) 和 (17) 可以清楚看出，仅横向方向存在物理谐振子，因此会自动构造出正范数的福克空间。在所有模式中，$n = 0$ 模式是特殊的，它给出了质壳条件

$$M_{\text{open}}^{2} = 2p^{+}p^{-} - p^{i}p^{i} = \frac{1}{2\alpha'}\sum_{m\neq 0}\alpha_{-m}^{i}\alpha_{m}^{i}, \tag{18}$$

for open strings, and

对应开弦，以及

$$M_{\text{closed}}^{2} = \frac{1}{\alpha'}\sum_{m\neq 0}\alpha_{-m}^{i}\alpha_{m}^{i} = \frac{1}{\alpha'}\sum_{m\neq 0}\tilde{\alpha}_{-m}^{i}\tilde{\alpha}_{m}^{i}. \tag{19}$$

We next employ the commutation relations (12) to bring the sums into normal-ordered form,

接下来我们利用对易关系 (12) 将求和整理为正规序形式,

$$\sum_{m\neq 0} \alpha^i_{-m}\alpha^i_m = \sum_{m>0}\left(\alpha^i_{-m}\alpha^i_m + \alpha^i_m\alpha^i_{-m}\right) = 2N - \frac{D-2}{12}, \tag{20}$$

where $N = \sum_{m>0} \alpha^i_{-m}\alpha^i_m$ and we have used zeta function regularisation to evaluate the infinite contribution $\sum_{m>0} m$. Although the latter step appears to be a somewhat ad hoc procedure, it is nevertheless justified by introducing a suitable counter-term proportional to a cosmological constant in the world-sheet action. Putting everything together, the mass formulae for open strings reads

其中 $N = \sum_{m>0} \alpha^i_{-m}\alpha^i_m$ ，我们使用了 ζ 函数正规化来计算无穷贡献 $\sum_{m>0} m$ 。尽管后一步看起来像是一种特殊的临时处理，但通过在世界面作用量中引入一个合适的、正比于宇宙学常数的抵消项，就可以证明这一步是合理的。整合所有结果后，开弦的质量公式为

$$M^2_{\text{open}} = \frac{1}{\alpha'}\left(N - \frac{D-2}{24}\right), \tag{21}$$

while for closed strings

而对于闭弦

$$M^2_{\text{closed}} = \frac{2}{\alpha'}\left(N + \tilde{N} - \frac{D-2}{12}\right), \tag{22}$$

and must be supplemented by the level-matching condition $N = \tilde{N}$.

并且必须补充能级匹配条件 $N = \tilde{N}$ 。

We are now ready to discuss the light spectrum, starting with open strings. The unique vacuum $|0\rangle$ is a space-time scalar wi first excitation is $\alpha^i_{-1}|0\rangle$ and transforms in the vectorial representation of the little group $\mathrm{SO}(D-2)$. This is compatible with Lorentz invariance if and only if this state is massless, which is the case only in $D = 26$ dimensions. This implies that the vacuum corresponds to a tachyonic state. The next levels $\alpha^i_{-2}\left|0\right\rangle$ and $\alpha^i_{-1}\alpha^j_{-1}\left|0\right\rangle$ are then massive with $M^2 = 1/\alpha'$. Although they are built out of the transverse oscillators, they can be re-organised into irreducible representations of the little group $SO(D-1)$, since a spin-2 representation of $\mathrm{SO}(D-1)$ can be decomposed into a spin-2 tensor plus a vector and a scalar of $\mathrm{SO}(D-2)$,

我们现在可以讨论轻子谱了,从开弦开始。唯一真空 $|0\rangle$ is a space-time scalar with mass $M^2 = -(D-2)/24\alpha'$. The | 的第一激发态是 $\alpha^i_{-1}|0\rangle$ ，它按小群 $\mathrm{SO}(D-2)$ 的矢量表示变换。当且仅当该态无质量时，这才符合洛伦兹不变性，而该情况仅发生在 $D = 26$ 维中。这说明真空对应一个快子态。下一能级 $\alpha^i_{-2}\left|0\right\rangle$ and $\alpha^i_{-1}\alpha^j_{-1}\left|0\right\rangle$ 是有质量的，满足 $M^2 = 1/\alpha'$ 。尽管它们由横振荡器构造而来，仍可以重新组织为小群 $SO(D-1)$ 的不可约表示，因为 $\mathrm{SO}(D-1)$ 的自旋 2 表示可以分解为 $\mathrm{SO}(D-2)$ 的自旋 2 张量、矢量和标量,

$$\frac{(D-1)D}{2} - 1 = \frac{(D-2)(D-1)}{2} - 1 + (D-2) + 1. \tag{23}$$

This is the Higgs mechanism applied to spin-2 fields. A similar pattern repeats for all higher levels.

这就是应用于自旋 2 场的希格斯机制。类似的模式在所有更高能级重复出现。

Turning to closed strings one has to properly tensor together left movers and right movers, while respecting level-matching. The unique vacuum $|0,\tilde{0}\rangle$ is again a spacetime scalar with mass $M^2 = -(D-2)/6\alpha'$. The first excited level is $\alpha^i_{-1}\tilde{\alpha}^j_{-1}|0,\tilde{0}\rangle$ which decomposes into the symmetric traceless, the anti-symmetric and the singlet representations of $\mathrm{SO}(D-2)$. As before, consistency with Lorentz symmetry requires that these states be massless, which occurs in $D = 26$ dimensions. Therefore, these states correspond to a massless spin-2 field, to be identified with the space-time graviton $G_{\mu\upsilon}$, a massless rank-2 anti-symmetric tensor $B_{\mu\upsilon}$, known as the Kalb-Ramond field, and a massless scalar Φ , known as the dilaton. The next level describes 104,976 massive degrees of freedom which, among others, include a massive spin-4 field.

转向闭弦，我们需要将左动模和右动模正确做张量积，同时满足能级匹配条件。唯一真空 $|0,\tilde{0}\rangle$ 仍是一个时空标量，质量为 $M^2 = -(D-2)/6\alpha'$ 。第一激发能级为 $\alpha^i_{-1}\tilde{\alpha}^j_{-1}|0,\tilde{0}\rangle$ ，它可分解为 $\mathrm{SO}(D-2)$ 的对称无迹、反对称和幺正表示。和之前一样，洛伦兹对称性的自洽性要求这些态都是无质量的，这仅在 $D = 26$ 维中成立。因此，这些态分别对应: 可识别为时空引力子的无质量自旋 2 场 $G_{\mu\upsilon}$ ，称作卡尔布-朗道场的无质量 2 阶反对称张量 $B_{\mu\upsilon}$ ，以及称作 dilation 的无质量标量 Φ 。下一能级描述了 104976 个有质量自由度，其中包含一个有质量自旋 4 场。

A convenient way to count the massive degrees of freedom of open and closed strings is to compute the one-loop vacuum energy. In quantum field theory, it is not a quantity of particular relevance since it is simply a function of the particle masses. Different is the situation in string theory, since it describes infinite particles associated to the various harmonics. As we shall see, it imposes non-trivial constraints on the consistency of the theory, especially in the case of fermionic strings. For a real scalar field, the quantity we wish to compute is

计算开弦和闭弦有质量自由度的一种简便方法是计算单圈真空能。在量子场论中，这个量并没有特别的相关性，因为它只是粒子质量的函数。弦论中的情况不同，因为弦论描述了与各谐波关联的无穷多粒子。我们将会看到，它对理论的自洽性给出了非平凡约束，费米弦的情况尤其明显。对于实标量场，我们想要计算的量是

$$\mathcal{L} = \int [\mathcal{D}\phi]\, e^{\int d^D x \frac{1}{2}\phi(\Box - m^2)\phi} = \mathrm{Det}^{-1/2}\left(-\Box + m^2\right), \tag{24}$$

or, rather, its logarithm. This can be most conveniently evaluated in the Schwinger representation

或者说，是它的对数。在施温格表示中可以最方便地计算这个量

$$\log \mathcal{Z} = -\frac{1}{2}\int_\varepsilon^\infty \frac{dt}{t}\,\mathrm{Tr}\, e^{-\pi t(-\Box + m^2)}, \tag{25}$$

where t is the proper time for a point particle moving along a circle, and ε regulates the UV divergence. In the following, we shall be cavalier and set ε to zero. The contribution of the box operator is universal and reads

其中 t 是沿圆周运动的点粒子的固有时，ε 用于规范紫外发散。接下来我们会简化处理，将 ε 设为零。箱算符的贡献是普适的，形式为

(26)

$$\operatorname{Tr} e^{\pi t \square} = \int d^D p \left\langle p \left| e^{-\pi p^2 t} \right| p \right\rangle = \int d^D p e^{-\pi p^2 t} \int d^D x \langle p \mid x \rangle \langle x \mid p \rangle$$

$$= \frac{V_D}{(2\pi)^D} \frac{1}{t^{D/2}},$$

so that

因此

$$\log \mathcal{Z} = -\frac{V_D}{2(2\pi)^D} \int_0^\infty \frac{dt}{t^{1+D/2}} \operatorname{Tr} e^{-\pi t m^2}. \tag{27}$$

It is straightforward to extend this expression to string theory, by simply replacing the mass of the particles m^2 by the mass operator M^2 . For open strings,

将这个表达式推广到弦论是很直接的，只需将粒子质量 m^2 替换为质量算符 M^2 。对于开弦，

$$\log \mathcal{Z} = -\frac{V_D}{2(2\pi)^D} \int_0^\infty \frac{dt}{t^{1+D/2}} \operatorname{Tr} e^{-\pi t \left(N - \frac{D-2}{24}\right)/\alpha'}. \tag{28}$$

Recall that N is the Hamiltonian of a system containing an infinite number of harmonic oscillators with frequencies $n = 1, 2, \ldots$ Therefore, setting $q = e^{-\pi t/\alpha'}$ and introducing the properly normalised ladder operators $a_n^i = \alpha_n^i/\sqrt{n}$ and $\left(a_n^i\right)^\dagger = \alpha_{-n}^i/\sqrt{n}$

回顾可知, N 是包含无穷多个频率为 $n = 1, 2, \ldots$ 的简谐振子的系统的哈密顿量。因此，令 $q = e^{-\pi t/\alpha'}$ ，并引入适当归一化的阶梯算符 $a_n^i = \alpha_n^i/\sqrt{n}$ 和 $\left(a_n^i\right)^\dagger = \alpha_{-n}^i/\sqrt{n}$

$$\operatorname{Tr} q^N = \operatorname{Tr} q^{\sum_{i=1}^{D-2} \sum_{n>0} n \left(a_n^i\right)^\dagger a_n^i} = \left[\prod_{n>0} \sum_{k\geq 0} q^{nk}\right]^{D-2} = \frac{1}{\prod_{n>0} \left(1-q^n\right)^{D-2}},$$

(29)

which indeed counts the number $P(n)$ of possible partitions of the total energy n into the various oscillators. Inserting this result into (28), one gets

它恰好可以统计总能量 n 分配给各个振子的可能分划数 $P(n)$ 。将该结果代入式 (28)，可得

$$\log \mathcal{Z} = -\frac{V_D}{2(2\pi)^D} \int_0^\infty \frac{dt}{t^{1+D/2}} \frac{1}{\eta(q)^{D-2}}, \tag{30}$$

written in terms of the Dedekind eta function

用戴德金 η 函数表示为

$$\eta(q) = q^{1/24} \prod_{n>0} (1 - q^n) . \tag{31}$$

As promised, upon Taylor expanding the integrand around $q = 0$, Eq. (30) encodes the degrees of freedom of open-string states at all mass levels. Indeed, introducing $Z = 1/\eta(q)^{24}$ in the case of $D = 26$ dimensions, one finds

正如之前所说，将被积函数在 $q = 0$ 附近泰勒展开后，式 (30) 给出了所有质量能级开弦态的自由度。实际上，对 $D = 26$ 维情况引入 $Z = 1/\eta(q)^{24}$ 后，可以得到

$$Z(q) = \sum_n d(n)\, q^n = \frac{1}{q} + 24 + 324q + 3200q^2 + \dots , \tag{32}$$

where $d(n)$ is the number of states at mass level $M^2 = n/\alpha'$. From this expansion we then recognise the real tachyon of the open string, the 24 d.o.f. of a massless vector, the 324 d.o.f.' s of a spin 2 field with mass $M^2 = 1/\alpha'$, and so on.

其中 $d(n)$ 是质量能级 $M^2 = n/\alpha'$ 的态数目。从该展开式中我们可以识别出开弦的实快子、零质量矢量的 24 个自由度、质量为 $M^2 = 1/\alpha'$ 的自旋 2 场的 324 个自由度，依此类推。

Moving to the closed string case, the mass operator receives contributions from both left-moving and right-moving oscillators, but physical states are those which obey the level-matching condition. Taking this into account, the closed string analogue of (28) is

转到闭弦情形，质量算符同时获得左行振子与右行振子的贡献，但只有满足能级匹配条件的态才是物理态。考虑到这一点，对应式 (28) 的闭弦表达式为

(33)

$$\begin{aligned} \log \mathcal{Z} &= -\frac{V_D}{2(2\pi)^D} \int_0^\infty \frac{dt}{t^{1+D/2}} \operatorname{Tr}\left[\delta_{N-\tilde{N}} e^{-2\pi t\left(N+\tilde{N}-\frac{D-2}{12}\right)/\alpha'}\right] \\ &= -\frac{V_D}{2(2\pi)^D} \int_0^\infty \frac{dt}{t^{1+D/2}} \int_{-1/2}^{1/2} ds \operatorname{Tr} q^{N-\frac{D-2}{24}}\, \bar{q}^{\tilde{N}-\frac{D-2}{24}} , \end{aligned}$$

where now $q = e^{2\pi i(s+it/\alpha')}$. A similar computation as in (29), yields

此时满足 $q = e^{2\pi i(s+it/\alpha')}$ 。通过与式 (29) 类似的计算，可得

$$\log \mathcal{Z} = -\frac{V_D}{2(2\pi)^D} \int_{-1/2}^{1/2} ds \int_0^\infty \frac{dt}{t^{1+D/2}} \frac{1}{\left(\eta(q)\,\bar{\eta}(\bar{q})\right)^{D-2}}, \tag{34}$$

where, by abuse of notation, we adopt the standard convention where the right-movers contribute with the anti-holomorphic function $\bar{\eta}(\bar{q}) \equiv (\eta(q))^*$. As before, Taylor expanding the integrand

其中，我们符合惯例滥用记号，采用标准约定: 右行振子贡献反全纯函数 $\bar{\eta}(\bar{q}) \equiv (\eta(q))^*$ 。和之前一样，对被积函数做泰勒展开

$$Z(q,\bar{q}) = \sum_{n,m} d(n,m)\, q^n \bar{q}^m = \frac{1}{q\bar{q}} + 24\left(\frac{1}{q}+\frac{1}{\bar{q}}\right) + 576 + 324\left(\frac{q}{\bar{q}}+\frac{\bar{q}}{q}\right)$$

$$+7776\,(q+\bar{q}) + 3200\left(\frac{q^2}{\bar{q}}+\frac{\bar{q}^2}{q}\right) + 76,800\left(q^2+\bar{q}^2\right) + 104,976 q\bar{q} + \ldots,$$

(35)

and, upon imposing level-matching to select the physical excitations $M^2 = 4n/\alpha'$, we recognise the real tachyon, 576 massless states comprising the graviton, the dilaton and the Kalb-Ramond field, and so on.

并且, 在施加能级匹配筛选出物理激发 $M^2 = 4n/\alpha'$ 后, 我们可以识别出实快子、包含引力子、dilaton 和卡尔布-朗道场在内的 576 个零质量态，依此类推。

Actually, Eq. (34) does not really take into account the extended nature of closed strings. The logic leading to (34) was to follow the analogy with quantum field theory of point particles, where the one-loop vacuum-to-vacuum amplitude corresponds to a world-line with the topology of a circle. The same diagram in closed strings should, thus, correspond to a doughnut, i.e., a worldsheet with the topology of a torus. As we shall see, the global properties of this worldsheet will impose non-trivial constraints on the consistency of a closed string vacuum.

实际上，式 (34) 并没有真正考虑闭弦的延展性质。推导式 (34) 的逻辑是类比点粒子量子场论: 其中单圈真空-真空振幅对应拓扑为圆的世界线，因此闭弦的同一费曼图应当对应甜甜圈形状，即拓扑为环面的世界面。我们将会看到，该世界面的整体性质会对闭弦真空的自洽性施加非平庸约束。

By analogy to the case of a circle, a flat torus can be constructed starting from two non-degenerate vectors ω_1, ω_2 in the complex plane, upon the identification $\Lambda : z \sim z + n\omega_1 + m\omega_2$, for all integers n, m . Therefore, the torus $T^2 = \mathbb{C}/\Lambda$ is an elementary cell, where opposite sides are identified. Contrary to the case of a circle, different choices of ω_1 and ω_2 do not necessarily define distinct torii. In fact, two choices of non-degenerate vectors ω_i and $\tilde{\omega}_i$ related by the linear transformation

和圆的情况类比，平直环面可以由复平面上两个非退化向量 ω_1, ω_2 构造得到，只要对所有整数 n, m 等同点 $\Lambda : z \sim z + n\omega_1 + m\omega_2$ 。因此，环面 $T^2 = \mathbb{C}/\Lambda$ 就是一个对边等同的基本胞元。和圆的情况不同，不同的 ω_1 和 ω_2 选择不一定定义不同的环面。事实上，若两组非退化向量 ω_i 和 $\tilde{\omega}_i$ 满足如下线性变换关系

$$\begin{pmatrix} \tilde{\omega}_1 \\ \tilde{\omega}_2 \end{pmatrix} = \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}, \tag{36}$$

define the same torus provided that $a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1$, which preserves the volume of the torus. Using the rotational and scaling symmetries, one may always bring one side of the parallelogram to lie on the real axis and normalise its length to one, so that the torus is identified by a complex number $\tau = \tau_1 + i\tau_2 = \omega_2/\omega_1$, known as the complex structure. The maps (36) act on τ as the fractional linear transformations

只要满足 $a, b, c, d \in \mathbb{Z}$ 和 $ad - bc = 1$ 就能定义同一个环面，该变换保持环面的体积不变。利用旋转对称性和标度对称性，我们总可以让平行四边形的一条边落在实轴上，并将其长度归一化为 1，因此环面可以由复数 $\tau = \tau_1 + i\tau_2 = \omega_2/\omega_1$ 刻画，这个复数称为复结构。映射 (36) 通过分式线性变换作用在 τ 上

$$\tilde{\tau} = \frac{a\tau + b}{c\tau + d} \tag{37}$$

The set of matrices in (36) are elements of the modular group $\mathrm{SL}(2;\mathbb{Z})$, generated by

(36) 中的矩阵集合都是模群 $\mathrm{SL}(2;\mathbb{Z})$ 的元素，该群由以下变换生成

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \ S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{38}$$

which act as translations, $T : \tau \to \tau + 1$, and inversions, $S : \tau \to -1/\tau$, on the complex structure.

它们分别对复结构作用为平移 $T : \tau \to \tau + 1$ 和反演 $S : \tau \to -1/\tau$

Returning to the partition function of closed strings (34), it is natural to identify $\tau = s + it/\alpha'$. Indeed, using the definition of q , the trace in (33) can be conveniently rewritten as

回到闭弦的配分函数 (34)，我们很自然可以认同 $\tau = s + it/\alpha'$ 。实际上，利用 q 的定义，(33) 中的迹可以方便地改写为

$$\mathrm{Tr}\left[e^{-2\pi\tau_2(N+\tilde{N}-2)} e^{2\pi i\tau_1(N-\tilde{N})}\right], \tag{39}$$

where $N+\tilde{N}-2$ is recognised as the two-dimensional Hamiltonian H associated to time-evolution, while $N - \tilde{N}$ is the two-dimensional momentum operator P generating translations along the spatial world-sheet direction. This has an interesting physical interpretation as a closed string state propagating for proper time τ_2 to form a cylinder, whose two ends are to be then glued together by the trace, after a relative rotation by an angle $2\pi\tau_1$. The resulting object is clearly a torus with metric $ds^2 = \tau_2^{-1}\left|d\sigma^1 + \tau d\sigma^0\right|^2$, which therefore justifies the aforementioned identification.

其中 $N+\tilde{N}-2$ 是对应时间演化的二维哈密顿量 H ，而 $N-\tilde{N}$ 是生成世界面空间方向平移的二维动量算符 P 。这有一个很直观的物理解释: 闭弦态经过固有时 τ_2 传播形成一个柱面，之后迹将柱面的两个端点相对旋转角度 $2\pi\tau_1$ 后粘合在一起。最终得到的显然是一个度规为 $ds^2 = \tau_2^{-1}\left|d\sigma^1 + \tau d\sigma^0\right|^2$ 的环面，这就为上述的认同提供了依据。

Since τ and $\tilde{\tau}$ related by (37) correspond to different parameterisations of the same worldsheet, consistency requires that the integrand of (34) be invariant under the action of the modular group. For the simple example of the closed bosonic string discussed so far, this is guaranteed by the modular properties of the Dedekind eta function

由于满足关系 (37) 的 τ 和 $\tilde{\tau}$ 对应同一个世界面的不同参数化，自洽性要求 (34) 的被积函数在模群的作用下不变。对于我们目前讨论的闭玻色弦简单例子，这一不变性由戴德金 η 函数的模性质保证

$$\eta(\tau+1) = e^{i\pi/12}\eta(\tau), \ \eta(-1/\tau) = \sqrt{-i\tau}\eta(\tau), \tag{40}$$

as first noticed by Shapiro [49]. In more general situations, however, we shall see that the requirement of modular invariance is non-trivial and actually provides the rationale for the construction of consistent string vacua. The integration over the s and t variables in (34) is now interpreted as a collective contribution of all world-sheet geometries with the topology of the torus. However, to avoid over-counting we should restrict the integration only over those complex structures that correspond to gauge-inequivalent torii. This effectively reduces the integration domain to

这一点最早由 Shapiro[49] 指出。但我们会看到，在更一般的情况下，模不变性的要求是非平凡的，它实际上是构造自洽弦真空的核心依据。现在我们可以将 (34) 中对 s 和 t 变量的积分，解释为所有具有环面拓扑的世界面几何的集体贡献。但为了避免重复计数，我们只需要对对应规范不等价环面的复结构积分，这可以有效地将积分区域约化为

$$\mathcal{F} = \mathbb{H}/\mathrm{SL}(2;\mathbb{Z}) = \left\{|\tau| \geq 1, -\frac{1}{2} \leq \tau_1 < \frac{1}{2}, \tau_2 > 0\right\}, \tag{41}$$

known as the fundamental domain of $\mathrm{SL}(2;\mathbb{Z})$, where $\mathbb{H}$ is the Poincaré upper-half plane, endowed with the hyperbolic metric.

该区域称为 $\mathrm{SL}(2;\mathbb{Z})$ 的基本域，其中 $\mathbb{H}$ 是配备双曲度量的庞加莱上半平面

A similar interpretation can be given for open strings, although in this case there is a single Schwinger parameter and modular invariance is no longer present. We defer this discussion to section ”Orientifolds,” since a deeper interpretation of the amplitude can be best achieved only after world-sheet fermions have been introduced.

开弦也可以给出类似的解释，只不过开弦只有一个施温格参数，不再存在模不变性。我们将这部分讨论推迟到“定向模”一节，因为只有引入世界面费米子之后，才能对振幅给出更深刻的诠释。

Two-Dimensional Conformal Field Theory

二维共形场论

The residual symmetry (13) that allowed us to quantise string theory in the light-cone actually plays a prominent role in the world-sheet description of the theory. Already the fact that the energy momentum tensor is traceless is an indication that the two-dimensional theory is invariant under conformal transformations. In general, the conformal group is the subgroup of general coordinate transformations, $\delta x^\mu = \xi^\mu(x)$, which leave the metric invariant up to an overall local rescaling

允许我们在光锥下对弦理论量子化的剩余对称性 (13)，实际上在该理论的世界面描述中占有重要地位。能量动量张量无迹这一事实就已经表明，二维理论在共形变换下不变。一般来说，共形群是一般坐标变换 $\delta x^\mu = \xi^\mu(x)$ 的子群，这些变换让度量只差一个整体局域标度因子保持不变

$$\delta g_{\mu\nu}(x) = \omega(x) g_{\mu\nu}(x). \tag{42}$$

In D dimensions, and specialising to the Minkowski metric this implies the relation

在 D 维中，对闵可夫斯基度量进行特殊化处理后，这给出关系

$$\partial_\mu \xi_\upsilon + \partial_\upsilon \xi_\mu = \frac{2}{D} \eta_{\mu\nu} \nabla \cdot \xi. \tag{43}$$

In $D > 2$ the general solution involves $\frac{1}{2}(D+1)(D+2)$ independent parameters, comprising the Poincaré transformations $\xi^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$, together with scale transformations $\xi^\mu = \lambda x^\mu$, and special conformal transformations $\xi^\mu = b^\mu x^2 - 2x^\mu b \cdot x$. In $D = 2$, instead, Eq. (43) reduces to the Cauchy-Riemann equations admitting an infinite number of solutions. These are nothing but the transformations $\sigma^\pm \to \sigma'^\pm(\sigma^\pm)$ that we discussed in section "The Bosonic String."

在 $D > 2$ 中，通解包含 $\frac{1}{2}(D+1)(D+2)$ 个独立参数，包括庞加莱变换 $\xi^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$ 、标度变换 $\xi^\mu = \lambda x^\mu$ 以及特殊共形变换 $\xi^\mu = b^\mu x^2 - 2x^\mu b \cdot x$ 。而在 $D = 2$ 中，式 (43) 退化为柯西-黎曼方程，存在无穷多解。这些正是我们在“玻色弦”一节讨论过的变换 $\sigma^\pm \to \sigma'^\pm(\sigma^\pm)$

In two-dimensional conformal field theories (CFTs) [50-53] (see [54-59] for an introduction), it is useful to work in Euclidean space and introduce the complex coordinates $z = e^{\tau+i\sigma}, \bar{z} = e^{\tau-i\sigma}$, which map the cylinder spanned by a closed string to the whole complex plane. In terms of these variables, the proper time τ becomes the radius of circles centred around the origin and, therefore, the generator of scale transformations on z is identified with the Hamiltonian of the system. This is referred to as radial quantisation. The conservation of the energy momentum tensor

在二维共形场论 (CFT)[50-53] 中 (入门介绍参见 [54-59])，在欧几里得空间中工作并引入复坐标 $z = e^{\tau+i\sigma}, \bar{z} = e^{\tau-i\sigma}$ 十分方便，这套坐标可将闭弦张成的柱面映射到整个复平面。在这些变量下，固有时 τ 就是原点为圆心的圆的半径，因此 z 上标度变换的生成元对应系统的哈密顿量。这被称为径向量子化。能量动量张量的守恒性

$$\bar{\partial} T_{zz} + \partial T_{z\bar{z}} = 0, \tag{44}$$

together with the scale invariance property $T_{z\bar{z}} = 0$, imply that $T_{zz} \equiv T(z)$ is (classically) an holomorphic function of z so that, for any holomorphic function $\xi(z)$,

加上标度不变性性质 $T_{z\bar{z}} = 0$ ，意味着 $T_{zz} \equiv T(z)$ (经典层面上) 是 z 的全纯函数，因此对任意全纯函数 $\xi(z)$,

$$T_\xi = \oint \frac{dz}{2\pi i} \xi(z) T(z), \tag{45}$$

generates the infinitesimal conformal transformations $\delta z = \xi(z)$, once the contour is taken to encircle the origin. Similar arguments can be made for the transformations $\delta \bar{z} = \bar{\xi}(\bar{z})$ generated by the anti-holomorphic energy-momentum tensor $\bar{T}(\bar{z}) \equiv T_{\bar{z}\bar{z}}$.

当围道环绕原点时，生成无穷小微分共形变换 $\delta z = \xi(z)$。对于由反全纯能量动量张量 $\bar{T}(\bar{z}) \equiv T_{\bar{z}\bar{z}}$ 生成的变换 $\delta\bar{z} = \bar{\xi}(\bar{z})$ 也可以得到类似结论。

On general grounds, an infinitesimal conformal transformation on an operator $\mathcal{O}(z,\bar{z})$ is given by the equal-time commutator $\left[T_\xi + \bar{T}_{\bar{\xi}}, \mathcal{O}(z,\bar{z})\right]$, which in QFT is to be suitably averaged in the path integral sense. However, path integrals return the time-ordered correlator and, therefore, in order to compute a commutator one needs to slightly deform the time of the Noether charges. In radial quantisation, this amounts to

一般来说，作用在算符 $\mathcal{O}(z,\bar{z})$ 上的无穷小微分共形变换由等时对易子 $\left[T_\xi + \bar{T}_{\bar{\xi}}, \mathcal{O}(z,\bar{z})\right]$ 给出，在量子场论中需要对其在路径积分意义下做适当平均。但路径积分给出时间序关联子，因此要计算对易子需要略微调整诺特荷的时间。在径向量子化中，这等价于

$$\left[T_\xi, \mathcal{O}(z,\bar{z})\right] = \oint \frac{dw}{2\pi i} \xi(w)\, T(w)\, \mathcal{O}(z,\bar{z}), \tag{46}$$

where the integration contour encircles z, and similarly for its anti-holomorphic counterpart. This integral is determined by the singular behaviour of the operator product expansion (OPE) of $T(w)\,\mathcal{O}(z,\bar{z})$ as $w \to z$. The latter depends on the properties of $\mathcal{O}$. For a tensorial operator $\mathcal{O}_{z\ldots\bar{z}\ldots}(z,\bar{z})$ of rank $(h,\bar{h})$, the transformation under $z \to z'$ and $\bar{z} \to \bar{z}'$ reads

其中积分围道环绕 z，其反全纯对应项同理。该积分由 $T(w)\,\mathcal{O}(z,\bar{z})$ 在 $w \to z$ 处的算符乘积展开 (OPE) 的奇异行为决定，而后者又依赖于 $\mathcal{O}$ 的性质。对于秩为 $(h,\bar{h})$ 的张量算符 $\mathcal{O}_{z\ldots\bar{z}\ldots}(z,\bar{z})$，其在 $z \to z'$ 和 $\bar{z} \to \bar{z}'$ 下的变换为

$$\mathcal{O}_{z\ldots\bar{z}\ldots}(z,\bar{z}) \to \left(\frac{dz'}{dz}\right)^h \left(\frac{d\bar{z}'}{d\bar{z}}\right)^{\bar{h}} \mathcal{O}_{z\ldots\bar{z}\ldots}(z',\bar{z}'), \tag{47}$$

which implies the OPE

由此可得算符乘积展开

$$\begin{aligned} T(w)\,\mathcal{O}(z,\bar{z}) &= \frac{h}{(w-z)^2}\mathcal{O}(z,\bar{z}) + \frac{1}{w-z}\partial\mathcal{O}(z,\bar{z}) + \text{ regular terms,} \\ \bar{T}(\bar{w})\,\mathcal{O}(z,\bar{z}) &= \frac{\bar{h}}{(\bar{w}-\bar{z})^2}\mathcal{O}(z,\bar{z}) + \frac{1}{\bar{w}-\bar{z}}\bar{\partial}\mathcal{O}(z,\bar{z}) + \text{ regular terms.} \end{aligned} \tag{48}$$

Fields that satisfy (48) are called primary fields of conformal weight $(h,\bar{h})$, but do not exhaust the fields present in a CFT. Secondary or descendant fields, which are derivatives of the primaries, have higher-order singularities in their OPEs with the energy-momentum tensor.

满足 (48) 的场称为共形重量为 $(h,\bar{h})$ 的主场，但主场并未穷尽共形场论 (CFT) 中的所有场。次级场也称派生场，是主场的导数，它们与能量动量张量的算符乘积展开包含更高阶的奇点。

We are now ready to apply these techniques to the Polyakov action and, for simplicity, we shall first consider a single free boson $X(z,\bar{z})$ described by the Lagrangian

现在我们可以将这些方法应用于波利雅科夫作用量，为简化起见，我们首先考虑由拉格朗日量描述的单个自由玻色子 $X(z,\bar{z})$

$$S = \frac{1}{2\pi\alpha'}\int d^2z\partial X\bar{\partial}X \tag{49}$$

with energy-momentum tensor

其能量动量张量为

$$T(z) = -\frac{1}{\alpha'}\partial X\partial X(z),\ \bar{T}(\bar{z}) = -\frac{1}{\alpha'}\bar{\partial}X\bar{\partial}X(\bar{z}), \tag{50}$$

and we suppress the explicit display of the normal ordering symbol. It is straightforward to work out the 2-point function

我们这里省略正规序符号的显式写出，直接计算两点函数即可得到

$$\langle X(z,\bar{z})X(w,\bar{w})\rangle = -\frac{\alpha'}{2}\log|z-w|^2, \tag{51}$$

which is ill-defined in the IR and, thus, X itself is not a conformal field. Its holomorphic derivative, however, is a primary field of conformal weight $(1,0)$, as shown by the OPE

该式在红外区域是不良定义的，因此 X 本身不是共形场。但正如算符乘积展开所示，它的全纯导数是共形重量为 $(1,0)$ 的主场

$$T(z)\partial X(w) = -\frac{2}{\alpha'}\partial X(z)\langle\partial X(z)\partial X(w)\rangle$$

$$= \frac{1}{(z-w)^2}\partial X(w) + \frac{1}{z-w}\partial^2X(w) + \text{ regular terms.} \tag{52}$$

Similarly, $\bar{\partial}X(\bar{z})$ is primary with weight $(0,1)$, while higher-order derivatives of X are not primary, as may be easily verified. There is, however, another important class of primary fields that may be constructed out of a free scalar, obtained by the normal ordered exponentials of the form $e^{ipX(z,\bar{z})}$, where p is a real parameter. Its OPE with the energy-momentum tensor

类似地，$\bar{\partial}X(\bar{z})$ 是共形重量为 $(0,1)$ 的主场，而 X 的高阶导数都不是主场，这一点很容易验证。不过，自由标量还可以构造出另一类重要的主场: 形如 $e^{ipX(z,\bar{z})}$ 的正规序指数，其中 p 是实参数。它与能量动量张量的算符乘积展开

$$T(z)e^{ipX(w,\bar{w})} = \frac{\alpha' p^2/4}{(z-w)^2}e^{ipX(w,\bar{w})} + \frac{1}{z-w}\partial\left(e^{ipX(w,\bar{w})}\right) + \text{ regular terms}$$

(53)

reveals that it has conformal weight $\left(\alpha' p^2/4, \alpha' p^2/4\right)$. Notice that e^{ipX} classically has zero scaling dimension and it is only at the quantum level that it acquires a non-trivial conformal weight.

表明它的共形重量为 $(\alpha' p^2/4, \alpha' p^2/4)$ 。注意经典层面 e^{ipX} 的标度维数为零，仅在量子层面它才获得非平凡的共形重量。

In the space of holomorphic functions, the monomials z^{n+1} constitute a basis, each corresponding to a different conformal transformation. The algebra of the corresponding generators

在全纯函数空间中，单项式 z^{n+1} 构成一组基，每个单项式对应一个不同的共形变换。对应生成元的代数

$$L_n = \oint \frac{dz}{2\pi i} z^{n+1} T(z), \tag{54}$$

requires the 2-point function of the energy-momentum tensor with itself. On dimensional grounds, since the naive conformal dimension of $T(z)$ is $(2, 0)$, one would expect

需要用到能量动量张量的自身两点函数。从量纲分析出发，由于 $T(z)$ 的朴素共形维数是 $(2, 0)$ ，我们可以预期

$$T(z)\,T(w) = \frac{c/2}{(z-w)^4} + \frac{2}{(z-w)^2} T(w) + \frac{1}{z-w} \partial T(w) + \text{ regular terms },$$

(55)

for some constant c . This form of the TT OPE can be shown to be true for a general CFT, and in the case of a single free scalar $c = 1$. A direct computation gives the Virasoro algebra

其中 c 为常数。这种形式的 TT 算符乘积展开可以证明对一般共形场论都成立，在单个自由标量 $c = 1$ 的情况下也成立。直接计算可得到维拉宿代数

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12} n \left(n^2 - 1\right) \delta_{n+m}. \tag{56}$$

The constant c is the central charge and, as we shall see, reflects a quantum violation of Weyl symmetry. From Eq. (56), it is also clear that only $L_0, L_{\pm 1}$ close into a finite-dimensional sub-algebra and, together with their right-moving counterparts $\bar{L}_0, \bar{L}_{\pm 1}$ they form the $\mathrm{SL}(2; \mathbb{C})$ subgroup of the conformal group.

常数 c 是中心荷，下文我们会看到它反映了外尔对称性的量子破缺。从式 (56) 可以清楚看出，只有 $L_0, L_{\pm 1}$ 自身构成有限维子代数，它们和右行对应项 $\bar{L}_0, \bar{L}_{\pm 1}$ 一起构成共形群的 $\mathrm{SL}(2; \mathbb{C})$ 子群。

We may build the Hilbert space of the CFT by defining suitable in- and out-states. Because of the map $z = e^{\tau + i\sigma}$ taking the cylinder to the sphere, the past $\tau \to -\infty$ corresponds to $z \to 0$ while the future $\tau \to \infty$ corresponds to $z \to \infty$. Given a primary field $\phi(z, \bar{z})$ of conformal weight $\left(h, \bar{h}\right)$, the in-state is defined as

我们可以通过定义合适的入态和出态来构造共形场论的希尔伯特空间。由于存在将圆柱映射到球面的 $z = e^{\tau + i\sigma}$ ，过去 $\tau \to -\infty$ 对应 $z \to 0$ ，而未来 $\tau \to \infty$ 对应 $z \to \infty$ 。给定共形权重为 $\left(h, \bar{h}\right)$ 的本原场 $\phi(z, \bar{z})$ ， 入态定义为

$$\left|\phi\right\rangle = \lim_{z, \bar{z} \to 0} \phi(z, \bar{z}) \left|0\right\rangle, \tag{57}$$

where $|0\rangle$ is the SL $(2;\mathbb{C})$ invariant vacuum annihilated by the Virasoro operators L_n , with $n \geq -1$. Clearly, the $T\phi$ OPE implies

其中 $|0\rangle$ 是被 Virasoro 算符 L_n 湮灭的 SL $(2;\mathbb{C})$ 不变真空，满足 $n \geq -1$ 。显然，$T\phi$ 算符乘积展开给出

$$L_0\,|\phi\rangle = h|\,\phi\rangle,\; L_n|\phi\rangle = 0 \;\text{ for } n \geq 1. \tag{58}$$

The action of raising operators $L_n, n < 0$ on $|\phi\rangle$ builds the descendant states associated to the primary field and define the Verma module of ϕ . As a simple example, take $\phi(z) = \partial X$, which admits the Laurent expansion

升算符 $L_n, n < 0$ 作用在 $|\phi\rangle$ 上构造出与本原场关联的后代态，并定义了 ϕ 的 Verma 模。举一个简单的例子，考虑 $\phi(z) = \partial X$ ，它可以做洛朗展开

$$i\partial X(z) = \sqrt{\alpha'/2}\sum_{n\in\mathbb{Z}}\frac{\alpha_n}{z^{n+1}}. \tag{59}$$

The corresponding in-state is, hence,

因此，对应的入态为

$$\lim_{z\to 0}\partial X(z)\,|0\rangle \propto \alpha_{-1}|\,0\rangle, \tag{60}$$

and is clearly identified with the first excited state (the vector) of the open string. The field $\phi(z,\bar{z})$ is known as the string vertex operator and plays a crucial role in string amplitudes. The definition of the out-states requires a careful analytic continuation to the Minkowski space cylinder (for a review, see [53,54,58,59]).

它显然对应开弦的第一激发态 (矢量态)。场 $\phi(z,\bar{z})$ 被称为弦顶点算符，在弦振幅中发挥着关键作用。出态的定义需要细致地解析延拓到闵氏空间圆柱 (综述参见文献 [53,54,58,59])。

The scalar field we have described so far does not exhaust the list of interesting CFTs with a free-field realisation. Another notable example important for string theory is the so-called $(h, 1-h)$ system [53]. This is made up of anti-commuting holomorphic primary fields b, c with conformal weight h and $1-h$, respectively. As we shall see, this system is rich enough to contain the reparametrisation ghosts as well as world-sheet fermions. The system is defined by the action

我们目前描述的标量场并未穷尽所有具有自由场实现的有趣共形场论。对弦理论而言另一个重要的著名例子是所谓的 $(h, 1-h)$ 系统 [53]。它由反对易的全纯本原场 b, c 构成，其共形权重分别为 h 和 $1-h$ 。我们将会看到，该系统足够丰富，既包含重新参数化鬼，也包含世界面费米子。该系统由如下作用量定义

$$S = \frac{1}{2\pi}\int d^2z d\bar{\partial}c, \tag{61}$$

from which one may extract the non-trivial correlator

由此可以提取出非平凡关联函数

$$\langle b(z)\, c(w)\rangle = \frac{1}{z-w}, \tag{62}$$

and the energy-momentum tensor

以及能量动量张量

$$T_{bc}(z) = -hb\partial c + (1-h)\,\partial bc. \tag{63}$$

From these expressions it is possible to extract the central charge of the bc system by simply evaluating the most singular term in $T(z)\,T(w)$. The result is

从这些表达式出发，我们只需计算 $T(z)\,T(w)$ 中最奇异的项就能得到 bc 系统的中心荷。结果为

$$c = 12h(1-h) - 2. \tag{64}$$

An analogous construction can be made for the analogous $\left(\bar{h}, 1-\bar{h}\right)$ system involving the anti-holomorphic fields $\bar{b}, \bar{c}$, with right-moving central charge $\bar{c} = 12\bar{h}\left(1-\bar{h}\right) - 2$.

对于包含反全纯场 $\bar{b}, \bar{c}$ 的 $\left(\bar{h}, 1-\bar{h}\right)$ 系统，可以构造类似的结构，其右行中心荷为 $\bar{c} = 12\bar{h}\left(1-\bar{h}\right)-2$ 。

Another interesting family of $(h, 1-h)$ systems is built out of commuting holomorphic primary fields β and γ . The change of statistics simply implies an extra minus sign in the correlator and in the central charge.

另一族有趣的 $(h, 1-h)$ 系统由对易的全纯本原场 β 和 γ 构造得到。统计性质的改变仅意味着在关联函数和中心荷中多一个负号。

String Perturbation Theory

弦微扰论

We return now to the study of the Polyakov action (6), seen as a two-dimensional QFT of D free scalars. The naive path integral quantisation

我们现在回到对波利雅科夫作用量 (6) 的研究，它可以看作 D 个自由标量构成的二维量子场论。朴素的路径积分量子化

$$\mathcal{Z} = \int [\mathcal{D}X]\,[\mathcal{D}g]\, e^{-S[X,g]} \tag{65}$$

is ill-defined since the action is clearly invariant under the local diffeomorphisms and Weyl rescalings. Therefore, the correct way [60] to compute the path integral is via the Faddeev-Popov procedure, which we now review in the simple case where the Riemann surface is taken to be the sphere (see [31] for a detailed exposition). Under reparametrisations, the "off-diagonal" components of the metric transform as

是不适定的，因为该作用量显然在局部微分同胚和外尔 rescaling 下不变。因此，计算路径积分的正确方法 [60] 是法捷耶夫-波波夫程序，我们现在就黎曼曲面取为球面的简单情况回顾这个方法 (详细阐述见 [31])。在重新参数化下，度规的「非对角」分量变换为

$$\delta g_{zz} = 2\nabla_z \xi_z,\ \delta g_{\bar{z}\bar{z}} = 2\nabla_{\bar{z}} \bar{\xi}_{\bar{z}}, \tag{66}$$

so that the appropriate conformal gauge fixing condition is $\delta g_{zz} = \delta g_{\bar{z}\bar{z}} = 0$. As usual, this is achieved by inserting

因此合适的共形规范固定条件是 $\delta g_{zz} = \delta g_{\bar{z}\bar{z}} = 0$ 。和通常一样，这通过插入

$$1 = \int [\mathcal{D}\gamma]\, \delta\left(g^{\gamma}_{zz}\right) \delta\left(g^{\gamma}_{\bar{z}\bar{z}}\right) \det\left(\frac{\delta g^{\gamma}_{zz}}{\delta\gamma}\right) \det\left(\frac{\delta g^{\gamma}_{\bar{z}\bar{z}}}{\delta\gamma}\right), \tag{67}$$

into the path integral, where g^{γ} denotes the new metric into which g is transformed by a reparametrisation γ and the integral over the group manifold. Since both the action and measures are invariant under the diffeomorphisms γ , the integration over the group manifold factorises and yields an irrelevant (infinite) volume factor. Furthermore, upon converting the determinants into Berezin integrals, one obtains

到路径积分中实现，其中 g^{γ} 表示经重新参数化 γ 变换后得到的新度规，g 是变换前的度规，此外还要对群流形积分。由于作用量和测度都在微分同胚 γ 下不变，群流形上的积分可以分离出来，得到一个不影响物理结果的 (无穷大) 体积因子。进一步将行列式转化为别列津积分后，我们得到

$$\mathcal{Z} = \int [\mathcal{D}\omega] \int [\mathcal{D}X]\, [\mathcal{D}b]\, [\mathcal{D}c]\, e^{-S_X[X] - S_{\text{ghost}}[b,c]}, \tag{68}$$

with

其中

$$S_X[X] = \frac{1}{2\pi\alpha'} \int d^2z \partial X^{\mu} \bar{\partial} X^{\nu} \eta_{\mu\nu},\ S_{\text{ghost}}[b,c] = \frac{1}{2\pi} \int d^2z \left(b\bar{\partial}c + \bar{b}\partial\bar{c}\right).$$

(69)

Here, b and c are the anti-commuting ghost fields associated to diffeomorphisms and carry conformal weight 2 and -1, respectively. They are a particular realisation of the $(h, 1-h)$ system discussed previously, with $h = 2$ and $c_{\text{ghost}} = -26$. The function $\omega(z, \bar{z})$ parametrises the "diagonal" components of the metric g , which is conformal to the two-dimensional Minkowski metric, $g_{z\bar{z}} = e^{\omega} \eta_{z\bar{z}}$. Classically, the integrand in (68) does not depend on ω and, again, its integral would give an irrelevant multiplicative (infinite) constant. However, this is not true quantum-mechanically, since in arbitrary dimension D , there is a Weyl anomaly proportional to $c_{\text{tot}} = c_X + c_{\text{ghost}}$. Although (68) may define a sensible theory even when c_{tot} is different than zero, critical string theory requires the vanishing of the Weyl anomaly and selects the dimension of spacetime $D = 26$. Indeed, in $D = 26$, the Virasoro algebra associated to the total energy-momentum tensor does not have a central extension, and conformal symmetry is exact at the quantum level.

此处 b 和 c 是对应微分同胚的反对易鬼场，分别带有共形权重 2 和-1。它们是之前讨论的 $(h, 1-h)$ 系统的一个具体实现，满足 $h=2$ 和 $c_{\text{ghost}}=-26$ 。函数 $\omega(z, \bar{z})$ 参数化了度规 g 的「对角」分量，g 共形于二维闵氏度规 $g_{z\bar{z}}=e^{\omega}\eta_{z\bar{z}}$ 。经典层面上，(68) 中的被积函数不依赖于 ω ，它的积分同样只会给出一个不影响物理结果的 (无穷大) 乘积常数。但在量子力学中这并不成立，因为在任意维度 D 下，都会存在正比于 $c_{\text{tot}}=c_X+c_{\text{ghost}}$ 的外尔反常。即使当 c_{tot} 不为零时，(68) 仍可以定义一个自洽的理论，但临界弦理论要求外尔反常消失，由此确定了时空维度 $D=26$ 。确实，在维度为 $D=26$ 时，总能量动量张量对应的维拉宿代数没有中心扩张，共形对称性在量子层面是严格成立的。

As in the case of Yang-Mills theory, S_X+S_{ghost} has a residual BRST symmetry generated by the Hermitian, nilpotent charge $Q_{\text{BRST}}=Q+\bar{Q}$, with

和杨-米尔斯理论的情况一样，S_X+S_{ghost} 存在由厄米幂零荷 $Q_{\text{BRST}}=Q+\bar{Q}$ 生成的剩余 BRST 对称性，满足

$$Q=\oint\frac{dz}{2\pi i}\left[c(z)\left(T_X(z)+\frac{1}{2}T_{\text{ghost}}(z)\right)+\frac{3}{2}\partial^2 c(z)\right], \tag{70}$$

and $\bar{Q}$ similarly given in terms of anti-holomorphic fields [53]. Note that the products in the integrand are assumed to be normal ordered, while the nilpotency of Q and $\bar{Q}$ is a consequence of the vanishing of the total central charge. Clearly, physical (gauge invariant) states $|\psi\rangle$ must be closed under the action of the BRST operator, $Q_{\text{BRST}}|\psi\rangle=0$. A trivial way to satisfy this requirement is to consider exact states, $|\chi\rangle=Q_{\text{BRST}}|\eta\rangle$, but these have zero norm and, therefore, decouple from physical processes,

且 $\bar{Q}$ 可以类似地用反全纯场表示 [53]。注意被积函数中的乘积默认是正规序乘积，而 Q 和 $\bar{Q}$ 的幂零性是总中心荷为零的结果。显然，物理 (规范不变) 态 $|\psi\rangle$ 必须在 BRST 算符作用下闭，即满足 $Q_{\text{BRST}}|\psi\rangle=0$ 。满足该要求的平凡情况是恰当态 $|\chi\rangle=Q_{\text{BRST}}|\eta\rangle$ ，但这些态的范数为零，因此会退耦于物理过程,

$$\left(\langle\psi_f|+\langle\chi_f|\right)S\left(|\psi_i\rangle+|\chi_i\rangle\right)=\langle\psi_f|S|\psi_i\rangle. \tag{71}$$

Therefore, if the S -matrix defines a unitary theory on the full Hilbert space, it is also unitary once restricted on the BRST cohomology. On general grounds, one expects that physical states do not contain any ghost excitation and, thus, should be proportional to the ghost vacuum. Notice that the zero modes c_0 and b_0 of the ghost fields commute with the Hamiltonian and satisfy the Clifford algebra $\{c_0, b_0\}=1$, which admits a two-dimensional representation $|\pm\rangle$ satisfying $b_0|+\rangle=|-\rangle$ and $c_0|-\rangle=|+\rangle$. It turns out that the correct definition of physical states involves $|-\rangle$, | so that $|\psi\rangle=\psi(0)|0\rangle\otimes|-\rangle$ and

因此，若 S 矩阵在整个希尔伯特空间上定义了一个幺正理论，将其限制在 BRST 上同调后，该理论仍然是幺正的。一般而言，我们预期物理态不包含任何鬼激发，因此应当正比于鬼真空。注意，鬼场的零模 c_0 和 b_0 与哈密顿量对易，并且满足允许二维表示 $|\pm\rangle$ satisfying $b_0|+\rangle=|-\rangle$ 和 $c_0|-\rangle=|+\rangle$ 的克利福德代数 $\{c_0, b_0\}=1$ 。可以发现，物理态的正确定义包含 $|-\rangle$, | ，因此 $|\psi\rangle=\psi(0)|0\rangle\otimes|-\rangle$ 且

$$Q\Big|\psi\rangle=\left(c_0\left(L_0^X-1\right)+\sum_{n>0}c_{-n}L_n^X\right)\Big|\psi\rangle=0, \tag{72}$$

yields the physical conditions (58) with conformal weight $h = 1$.

给出共形权重为 $h = 1$ 的物理条件 (58)。

A generic physical state in string theory carries spacetime momentum p_μ , which is injected by the operator $e^{ip\cdot X}$. Indeed, from (59) it is clear that the momentum operator α_0^μ acts on the state as

弦理论中任意一个一般物理态都携带时空动量 p_μ ，该动量由算符 $e^{ip\cdot X}$ 引入。确实，从式 (59) 可以清楚看出，动量算符 α_0^μ 对态的作用为

$$\alpha_0^\mu e^{ip\cdot X}\Big|0\rangle = i\sqrt{2/\alpha'}\oint \frac{dz}{2\pi i}\partial X^\mu(z)\, e^{ip\cdot X(0)}\Big|0\rangle = \sqrt{\alpha'/2}p^\mu|0\rangle. \tag{73}$$

The BRST condition then implies that the state created by the vertex operator $e^{ip\cdot X}$ is a tachyon with $p^2 = 4/\alpha'$, which coincides with the vacuum discussed previously in the context of light-cone quantisation of closed strings. The first excited states correspond to the vertex operator $V(\zeta, p) = \zeta_{\mu\nu}\partial X^\mu\bar{\partial}X^\nu e^{ip\cdot X}$, which has conformal dimension $(1,1)$ provided $p^2 = 0$. The BRST condition further implies transversality, $\zeta_{\mu\nu}p^\mu = \zeta_{\mu\nu}p^\nu = 0$, so that it describes the massless graviton, the dilaton and the Kalb-Ramond field. The construction of physical massive states proceeds in a similar fashion.

BRST 条件由此推出，由顶点算符 $e^{ip\cdot X}$ 产生的态是满足 $p^2 = 4/\alpha'$ 的快子，这与之前闭弦光锥量子化讨论的真空一致。第一激发态对应顶点算符 $V(\zeta, p) = \zeta_{\mu\nu}\partial X^\mu\bar{\partial}X^\nu e^{ip\cdot X}$ ，当 $p^2 = 0$ 时它具有共形维度 $(1,1)$ 。BRST 条件进一步要求横截性 $\zeta_{\mu\nu}p^\mu = \zeta_{\mu\nu}p^\nu = 0$ ，因此该态描述无质量引力子、dilaton 和卡尔布-拉蒙德场。有质量物理态的构造也遵循类似方式。

Until now, we have considered the propagation of strings on a simple Minkowski spacetime and it is natural to ask what happens when the background geometry is non-trivial [61-68]. This implies that the Minkowski metric $\eta_{\mu\nu}$ in the Polyakov action should be replaced by a general pseudo-Riemannian metric $G_{\mu\nu}(X)$. Actually in string theory one has the freedom to deform the background by introducing non-trivial configurations for the Kalb-Ramond field $B_{\mu\nu}(X)$ and the dilaton $\Phi(X)$. This gives rise to a non-trivial sigma model described by

迄今为止, 我们仅讨论了弦在简单闵氏时空中的传播, 很自然会提出问题: 当背景几何非平凡时会发生什么 [61-68]。这意味着波利亚科夫作用量中的闵氏度规 $\eta_{\mu\nu}$ 应当替换为一般伪黎曼度规 $G_{\mu\nu}(X)$ 。实际上在弦理论中，我们可以通过引入非平凡构型来形变背景，这些构型对应卡尔布-拉蒙德场 $B_{\mu\nu}(X)$ 和 dilaton 场 $\Phi(X)$ ，由此得到了一个由下式描述的非平凡 sigma 模型:

$$S = -\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\left(\sqrt{-g}g^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu}(X) + \varepsilon_{ab}\partial_a X^\mu\partial_b X^\nu B_{\mu\nu}(X)\right.$$

$$\left.-\alpha'\sqrt{-g}R^{(2)}\Phi(X)\right),$$

(74)

where the last term is the coupling of the dilaton to the two-dimensional Ricci scalar. An important point is that not all backgrounds admit a consistent propagation of strings, due to the occurrence of the Weyl anomaly at the quantum level. It is possible to show that consistent backgrounds are those which yield vanishing beta functions for the three above action terms. A long and tedious computation yields the conformal invariance conditions

其中最后一项是 dilaton 与二维里奇标量的耦合。一个重要的点是，由于量子层次会出现外尔反常，并非所有背景都允许弦一致传播。可以证明，只有能让上述三个作用量项的 β 函数都为零的背景才是自洽的。经过冗长乏味的计算，我们得到共形不变性条件为

$$
\begin{aligned}
0 &= \beta^{G}_{\mu\upsilon} = R_{\mu\upsilon} + \frac{1}{4} H_{\mu}{}^{\lambda\rho} H_{\upsilon\lambda\rho} - 2\nabla_{\mu}\nabla_{\upsilon}\Phi + \mathcal{O}\left(\alpha'\right), \\
0 &= \beta^{B}_{\mu\nu} = \nabla_{\lambda} H^{\lambda}{}_{\mu\nu} - 2\nabla_{\lambda}\Phi H^{\lambda}{}_{\mu\nu} + \mathcal{O}\left(\alpha'\right), \\
0 &= \beta^{\Phi} = 4\nabla_{\lambda}\Phi\nabla^{\lambda}\Phi - 4\nabla_{\lambda}\nabla^{\lambda}\Phi + R + \frac{1}{12} H_{\lambda\rho\sigma} H^{\lambda\rho\sigma} + \mathcal{O}\left(\alpha'\right),
\end{aligned}
\tag{75}
$$

where ∇ is the standard covariant derivative acting on tensors and $H_{\mu\upsilon\rho} = \partial_{\mu} B_{\upsilon\rho} + \partial_{\upsilon} B_{\rho\mu} + \partial_{\rho} B_{\mu\upsilon}$ is the field strength for the Kalb-Ramond field. Therefore, only backgrounds which satisfy these conditions are conformal and give rise to consistent string theories. It is not a coincidence that these equations correspond to the Euler-Lagrange equation emanating from the effective action

其中 ∇ 是作用在张量上的标准协变导数，$H_{\mu\upsilon\rho} = \partial_{\mu} B_{\upsilon\rho} + \partial_{\upsilon} B_{\rho\mu} + \partial_{\rho} B_{\mu\upsilon}$ 是卡尔布-拉蒙德场的场强。因此，只有满足这些条件的背景才具有共形不变性，能给出自洽的弦理论。这些方程恰好就是有效作用量导出的欧拉-拉格朗日方程，这并非巧合:

$$
S_{\text{eff}} = -\frac{1}{2\kappa^2} \int d^{26}x \sqrt{-G} e^{-2\Phi} \left(R - 4\nabla_{\mu}\Phi\nabla^{\mu}\Phi + \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + \mathcal{O}\left(\alpha'\right) \right),
$$

(76)

where, on dimensional grounds, κ^2 is proportional to $\left(\alpha'\right)^{12}$. The $\mathcal{O}\left(\alpha'\right)$ terms contain higher derivative couplings and describe string corrections to Einstein gravity. The overall factor $e^{-2\Phi}$ is a characteristic of string theory and originates from the way the dilaton couples to the two-dimensional Ricci tensor in (74). Although technically this factor could be removed by a suitable Weyl rescaling of the spacetime metric so that one recovers the canonical Einstein-Hilbert term, it is actually useful in the sense that it identifies the quantum corrections to the effective action. Indeed, expanding (74) around a constant dilaton background, which trivially solves the beta function equations (75), the path integral weights each world-sheet topology by the factor $e^{-\langle\Phi\rangle\chi(\sum)}$, with $\chi\left(\sum\right)$ being the Euler characteristic of the world-sheet $\sum$. This way, we can identify $e^{\langle\Phi\rangle}$ with the string coupling constant g_s and the path integral implies a sum over all topologies of increasing Euler number. For closed oriented strings, $\chi\left(\sum\right) = 2 - 2g$ with the genus g counting the number of handles of $\sum$. In open (and possibly unoriented) strings, the Riemann surfaces involve boundaries (and possibly cross-caps) so that the Euler number becomes $\chi\left(\sum\right) = 2 - 2g - b - c$ where b counts the number of boundaries and c the number of cross-caps, not to be confused with the b, c ghosts. This way, the topological expansion parallels the loop expansion of QFT, where χ is the stringy analogue of the loop order. As a result, the effective action S_{eff} is a double expansion in both α' which contains the corrections due to the extended nature of strings, and in g_s which incorporates quantum effects. In this sense, the terms weighted by $e^{-2\Phi}$ in (76) are the tree-level contributions associated to the world-sheet topology of the sphere.

根据量纲分析，此处 κ^2 正比于 $(\alpha')^{12}$ 。$\mathcal{O}(\alpha')$ 项包含更高阶导数耦合，描述对爱因斯坦引力的弦修正。整体因子 $e^{-2\Phi}$ 是弦理论的特征，它来源于 (74) 式中 dilaton 耦合二维里奇张量的方式。尽管从技术上讲，我们可以通过对时空度量做合适的外尔标度变换消去这个因子，从而得到标准的爱因斯坦-希尔伯特项，但这个因子实际上很有用，因为它标识了有效作用量的量子修正。确实，在常数 dilaton 背景下展开 (74) 式——该背景平凡满足 β 函数方程 (75)——路径积分对每个世界面拓扑赋予权重因子 $e^{-\langle\Phi\rangle\chi(\Sigma)}$ ，其中 $\chi(\sum)$ 是世界面 $\sum$ 的欧拉示性数。由此我们可以将 $e^{\langle\Phi\rangle}$ 等同于弦耦合常数 g_s ，路径积分隐含了对所有欧拉数递增的拓扑求和。对于闭定向弦，有 $\chi(\sum) = 2-2g$ ，其中亏格 g 计数了 $\sum$ 的柄数。对于开弦 (也可能是非定向弦)，黎曼曲面包含边界 (也可能包含十字帽)，因此欧拉数变为 $\chi(\sum) = 2-2g-b-c$ ，其中 b 计数边界数目，c 计数十字帽数目，注意不要和 b, c 鬼场混淆。这种拓扑展开平行于量子场论的圈展开，其中 χ 就是弦论中对应圈阶的量。因此，有效作用量 S_{eff} 是关于 α' 和 g_s 的双重展开: α' 对应弦延展性质带来的修正，g_s 则包含量子效应。从这个意义上说，(76) 式中被 $e^{-2\Phi}$ 加权的项就是对应球面世界面拓扑的树图贡献。

At fixed genus g , a generic scattering amplitude of interest will involve N insertions of vertex operators associated to the external legs, whose positions typically have to be integrated over the corresponding Riemann surface. It turns out that for $g = 0, 1$ there is a residual symmetry which is not fixed by our choice of conformal gauge. This is known as the Conformal Killing Group (CKG) and is $\mathrm{SL}(2;\mathbb{C})$ in the case of the sphere, and translations in the case of the torus.

固定亏格 g 时，我们关心的一般散射振幅会包含 N 个与外腿关联的顶点算符插入，这些插入点的位置通常需要在对应黎曼曲面上积分。可以证明，对于 $g = 0, 1$ ，共形规范固定后还存在残余对称性。这就是我们所知的共形 Killing 群 (CKG): 对于球面它是 $\mathrm{SL}(2;\mathbb{C})$ ，对于环面它是平移群。

For higher genera the CKG is trivial. This residual symmetry is reflected in the presence of zero modes for the c -ghost, and there are $C_0 = 3$ of them on the sphere, only $C_1 = 1$ on the torus, while $C_{g>1} = 0$. Also the b -ghost can have non-trivial zero modes which count the number of conformally invariant complex moduli that describe a surface $\sum$ of genus g . The Riemann-Roch theorem relates the number B_g of the b -ghost zero modes to C_g by

对于高亏格曲面，共形 Killing 群是平凡的。这种残余对称性体现为 c 鬼场存在零模: 球面上共有 $C_0 = 3$ 个这样的零模，环面上仅有 $C_1 = 1$ 个，而 $C_{g>1} = 0$ 。此外 b 鬼场也可以有非平凡零模，其数目等于描述亏格 g 曲面 $\sum$ 的共形不变复模的数目。黎曼-罗赫定理将 b 鬼场零模的数目 B_g 与 C_g 关联起来:

$$B_g - C_g = 3g - 3, \tag{77}$$

so that there are no b zero modes on the sphere, which indeed has no moduli, one b zero mode on the torus which is characterised by its complex structure τ , and $3g-3b$ -zero modes on a genus- g surface. Given that b, c are Grassmann variables, the path integral is non-trivial only provided a suitable number of c and b fields are inserted. For instance, on the sphere, the $\mathrm{SL}(2;\mathbb{C})$ symmetry allows one to fix the positions of three vertex operators, conventionally chosen as $0, 1, \infty$, while dressing each of them with a c -ghost. Diffeomorphism invariance then requires that the positions of the remaining $N-3$ vertex operators be integrated over the sphere.

因此，球面上不存在 b 零模，球面确实没有模；环面上存在一个 b 零模，由其复结构 τ 表征；亏格为 g 的曲面上存在 $3g-3b$ 个零模。由于 b, c 是格拉斯曼变量，路径积分只有在插入了适当数量的 c 场和 b 场时才是非平凡的。例如，在球面上，$\mathrm{SL}(2;\mathbb{C})$ 对称性允许我们固定三个顶点算符的位置，按照惯例固定为 $0, 1, \infty$ ，并给每个顶点算符配备一个 c 鬼场。微分同胚不变性进一步要求，其余 $N-3$ 个顶点算符的位置需要在整个球面上积分。

As an example, we can study the simplest non-trivial scattering amplitude on the sphere involving four external tachyons of incoming momenta p_i . According to the previous discussion, we fix the positions of three of them $z_{1,2,3}$ accompanying their vertex operators by c -ghosts, while the fourth one does not carry any ghost and its position z_4 is integrated over the sphere,

举个例子，我们可以研究球面上最简单的非平凡散射振幅，该振幅涉及四个入射动量为 p_i 的外快子。根据前文的讨论，我们固定其中三个顶点算符的位置为 $z_{1,2,3}$ ，并为它们配备 c 鬼场，而第四个顶点算符不携带鬼场，其位置 z_4 需要在整个球面上积分。

$$\mathcal{A}_c(p_i) \sim g_s^2 \langle c(z_1)\bar{c}(\bar{z}_1) e^{ip_1\cdot X(z_1,\bar{z}_1)} c(z_2)\bar{c}(\bar{z}_2) e^{ip_2\cdot X(z_2,\bar{z}_2)} c(z_3)\bar{c}(\bar{z}_3) e^{ip_3\cdot X(z_3,\bar{z}_3)}$$

$$\times \int d^2 z_4 e^{ip_4\cdot X(z_4,\bar{z}_4)}.$$

(78)

The g_s dependence of the amplitude can be justified as follows. This is a tree-level diagram involving the topology of a sphere, and is therefore weighted by g_s^{-2} . For each external leg, there is a cylinder (the propagator) which connects the scattered states to the sphere, therefore introducing a boundary which by the previous argument is weighted by g_s . Upon mapping the sphere to the complex plane, each external leg shrinks to a puncture, where the vertex operator is inserted. In the case at hand we have four external states, so that the overall factor is $g_s^{-2}g_s^4 = g_s^2$.

振幅对 g_s 的依赖关系可以按如下方式推导: 这是一个球面拓扑的树图，因此权重为 g_s^{-2} 。每个外腿都对应一个连接散射态与球面的柱体 (即传播子)，因此引入了一个边界，根据前文的论证，该边界的权重为 g_s 。将球面映射到复平面后，每个外腿收缩为一个穿刺点，顶点算符就插入在该位置。在我们讨论的这个情况中共有四个外态，因此整体因子为 $g_s^{-2}g_s^4 = g_s^2$.

The correlators involving the holomorphic and anti-holomorphic ghosts factorise and can be computed independently. Requiring that the conformal symmetry generated by $c(z)\partial$ be regular at infinity implies that only the three generators $c_{-1}z^2\partial, c_0 z\partial$ and $c_1\partial$ corresponding to the $\mathrm{SL}(2;\mathbb{C})$ subgroup contribute. Taking into account the fact that the operators c_n anti-commute, one finds

涉及全纯鬼场和反全纯鬼场的关联函数可以因子化，能够独立计算。要求由 $c(z)\partial$ 生成的共形对称性在无穷远处正则，意味着只有对应 $\mathrm{SL}(2;\mathbb{C})$ 子群的三个生成元 $c_{-1}z^2\partial, c_0 z\partial$ 和 $c_1\partial$ 有贡献。考虑到算符 c_n 反对易，可以得到

$$\langle c(z_1)\, c(z_2)\, c(z_3)\rangle = z_{12}z_{13}z_{23}, \tag{79}$$

and similarly for the anti-holomorphic ghosts. Here and in the following, we adopt the standard notation $z_{ij} = z_i - z_j$. Evaluating the X correlators also produces the standard momentum conserving delta function so that

反全纯鬼场的结果类似。在下文中，我们采用标准记号 $z_{ij} = z_i - z_j$ 。计算 X 关联函数也会得到标准的动量守恒 δ 函数，因此

$$\mathcal{A}_c(p_i) \sim g_s^2 \delta^{(26)}(p_1 + p_2 + p_3 + p_4) |z_{12} z_{13} z_{23}|^2 \int d^2 z_4 \prod_{i<j=1}^{4} |z_{ij}|^{\alpha' p_i \cdot p_j}.$$

(80)

It is conventional to set $z_{1,2,3}$ to $0, 1, \infty$, respectively, and introduce the Mandelstam variables $s = -(p_1 + p_2)^2, t = -(p_1 - p_3)^2$, and $u = -(p_1 - p_4)^2$. Using the mass-shell condition $p_i^2 = -4/\alpha'$ one arrives at the celebrated Shapiro-Virasoro amplitude [69, 70]

按照惯例，我们分别将 $z_{1,2,3}$ 设为 $0, 1, \infty$ ，并引入曼德尔斯坦变量 $s = -(p_1 + p_2)^2, t = -(p_1 - p_3)^2$ 和 $u = -(p_1 - p_4)^2$ 。利用在壳条件 $p_i^2 = -4/\alpha'$ ，我们可以得到著名的夏皮罗-维拉索罗振幅 [69, 70]

$$\mathcal{A}_c(p_i) \sim g_s^2 \delta^{(26)}(p_1 + p_2 + p_3 + p_4) \frac{\Gamma\left(-1 - \frac{\alpha' s}{4}\right) \Gamma\left(-1 - \frac{\alpha' t}{4}\right) \Gamma\left(-1 - \frac{\alpha' u}{4}\right)}{\Gamma\left(2 + \frac{\alpha' s}{4}\right) \Gamma\left(2 + \frac{\alpha' t}{4}\right) \Gamma\left(2 + \frac{\alpha' u}{4}\right)}.$$

(81)

In the field theory limit, this single amplitude describes the three processes associated to the s, t and u channels, so that crossing symmetry is built in. Using the properties of the Γ functions, one may show that $\mathcal{A}_c(p_i)$ enjoys the Regge behaviour

在场论极限下，单个振幅就可以描述对应 s, t 道和 u 道的三个过程，因此交叉对称性是内禀的。利用 Γ 函数的性质，可以证明 $\mathcal{A}_c(p_i)$ 满足雷杰行为

$$\mathcal{A}_c(p_i) \propto s^{2+\alpha' t/2} \frac{\Gamma\left(-1 - \frac{\alpha' t}{4}\right)}{\Gamma\left(2 + \frac{\alpha' t}{4}\right)}, \tag{82}$$

for large s and fixed t , and is exponentially suppressed

当 s 很大、t 固定时，振幅呈指数压低

$$\mathcal{A}_c(p_i) \propto e^{-\frac{\alpha'}{2}(s \log s + t \log t + u \log u)}, \tag{83}$$

for $s, t, u \to \infty$.

对于 $s, t, u \to \infty$ 。

For completeness, we can briefly discuss the scattering of four open string tachyons. The endpoints of open strings trace boundaries upon their propagation, so that the relevant Riemann surface is now a disk with vertex operators attached to its boundary. Conformal transformations map this surface to the upper complex plane with vertex operators inserted on the real axis. The corresponding amplitude reads

为了内容完整，我们可以简要讨论四个开弦快子的散射。开弦的端点在传播过程中会勾勒出边界，因此对应的黎曼曲面是边界上附着顶点算符的圆盘。共形变换可将该曲面映射为实轴上插入顶点算符的上复平面，对应的振幅可写为

$$\mathcal{A}_o(p_i) \sim g_s \left\langle c(x_1)\, e^{ip_1\cdot X(x_1)} c(x_2)\, e^{ip_2\cdot X(x_2)} c(x_3)\, e^{ip_3\cdot X(x_3)} \int dx_4 e^{ip_4\cdot X(x_4)} \right\rangle . \tag{84}$$

The calculation of the correlators follows a similar procedure and, by fixing $x_{1,2,3}$ to $0, 1, \infty$, respectively, and integrating x_4 over $[0, 1]$ one obtains

关联函数的计算遵循相似的流程，分别将 $x_{1,2,3}$ 固定为 $0, 1, \infty$ ，再对 x_4 在 $[0, 1]$ 上积分后可得

$$\mathcal{A}_o(p_i) \sim g_s \delta^{(26)}(p_1 + p_2 + p_3 + p_4)\, B(-1 - \alpha' s, -1 - \alpha' t)\,, \tag{85}$$

where $B(p, q)$ is the Euler beta function. Clearly this is only one out of six possible choices of ordering the positions x_i . Summing over all possibilities, one obtains the celebrated Veneziano amplitude [1]

其中 $B(p, q)$ 是欧拉贝塔函数。显然，这只是位置 x_i 六种可能排序中的一种。对所有可能性求和后，就得到了著名的韦内齐亚诺振幅 [1]

$$\mathcal{A}_o(p_i) \sim g_s \delta^{(26)}(p_1 + p_2 + p_3 + p_4)\, [B(-1 - \alpha' s, -1 - \alpha' t) \tag{86}$$

$$+B(-1 - \alpha' s, -1 - \alpha' u) + B(-1 - \alpha' t, -1 - \alpha' u)]\ ,$$

which is fully crossing symmetric.

该振幅具有完全的交叉对称性。

Higher genus amplitudes can be computed following a similar pattern, but involve the CFT correlators on genus- g Riemann surfaces, as well as the integration over the corresponding moduli. In general, this can quickly become involved and, in the following, we shall focus on the genus one vacuum energy [71].

亏格更高的振幅可以遵循同样的模式计算，但会涉及亏格- g 黎曼曲面上的共形场论关联函数，以及对应模空间上的积分。一般来说，这个过程会很快变得复杂，因此在下文中我们将聚焦于亏格为 1 的真空能 [71]。

In this case, the path integral over the worldsheet metric reduces to a finite dimensional integral of the complex structure τ of the torus, parametrising gauge inequivalent metrics. The CKG of the torus is Abelian and contains two translations. Its volume is finite and given by $\int d^2z = \tau_2$ and, thus,

在这种情况下，世界面度规的路径积分会约化为环面复结构 τ 的有限维积分，复结构负责参数化不等价规范的度规。环面的共形基灵群 (CKG) 是阿贝尔群，包含两个平移变换。其体积有限，为 $\int d^2 z = \tau_2$ ，因此

$$\int \frac{[\mathcal{D}g]}{\mathrm{vol}\,(\mathrm{CKG})} \to \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \tag{87}$$

with $\mathcal{F}$ being the fundamental domain (41). Notice that for scattering amplitudes, one instead fixes the position of one vertex operator and, therefore, the τ_2 factor associated to vol(CKG) is now absent. One is left to perform the Gaussian integrals over the scalar fields X^μ and the reparametrisation ghosts.

其中 $\mathcal{F}$ 是基本域 (41)。注意, 对于散射振幅, 我们会额外固定一个顶点算符的位置, 因此与 vol(CKG) 相关的 τ_2 因子不再存在，接下来只需要对标量场 X^μ 和重参数化鬼完成高斯积分。

Let us start with the contribution of a single scalar X with periodic boundary conditions along both cycles of the worldsheet torus with metric

我们先从单个标量 X 的贡献开始讨论，该标量在带度规的世界面环面的两个闭路上都满足周期性边界条件

$$g_{ab} = \frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}. \tag{88}$$

We can now expand X onto the orthonormal basis of eigenmodes $\phi_{m,n}\left(\sigma^1, \sigma^2\right)$ of the Laplace operator $\Box$ on the torus,

我们现在可以将 X 展开在环面上拉普拉斯算符 $\Box$ 的本征模 $\phi_{m,n}\left(\sigma^1, \sigma^2\right)$ 的标准正交基上,

$$X = \sum_{m,n} c_{m,n} \phi_{m,n} \tag{89}$$

where $\Box\phi_{m,n} = -\lambda_{m,n}\phi_{m,n}$ with

其中 $\Box\phi_{m,n} = -\lambda_{m,n}\phi_{m,n}$ 满足

$$\lambda_{m,n} = \frac{\pi^2}{\tau_2^2}|m - \tau n|^2. \tag{90}$$

Notice that the zero mode $\phi_{0,0}$ is actually constant, due to the periodicity conditions, and it is fixed by the normalisation condition $\left\|\phi_{0,0}\right\| = 1$ to be $\phi_{0,0} = \tau_2^{-1/2}$. The path integral over X now turns into an integral over the Fourier modes $c_{m,n}$. In particular, the integral over the zero mode is

注意，由于周期性条件，零模 $\phi_{0,0}$ 实际上是常数，根据归一化条件 $\left\|\phi_{0,0}\right\| = 1$ 被固定为 $\phi_{0,0} = \tau_2^{-1/2}$ 。对 X 的路径积分现在转化为对傅里叶模 $c_{m,n}$ 的积分。具体来说，零模的积分是

$$\int dc_{0,0} = \int \frac{dx}{\phi_{0,0}} = L\sqrt{\tau_2}, \tag{91}$$

where L is the infinite linear volume spanned by the centre of mass x of the string. The integral over the remaining Fourier modes produces $\left(\det{}'\square\right)^{-1/2} = \prod'_{m,n} \lambda_{m,n}^{-1/2}$, where the primes indicate that the zero eigenvalue $\lambda_{0,0}$ is excluded. This infinite product is clearly divergent and must be carefully defined using zeta function regularisation. The main ingredients needed for this calculation are

其中 L 是弦质心 x 张成的无限线性体积。对剩余傅里叶模的积分得到 $\left(\det{}'\square\right)^{-1/2} = \prod'_{m,n} \lambda_{m,n}^{-1/2}$ ，其中撇号表示排除零本征值 $\lambda_{0,0}$ 。这个无穷乘积显然发散，必须利用 zeta 函数正则化来仔细定义。该计算所需的核心要素为

$$\prod_{n\neq 0} a = a^{-2\zeta(0)}, \ \prod_{n>0} n^{\alpha} = e^{-\alpha\zeta'(0)}, \ \prod_{n>0}\left(1-\frac{a^2}{n^2}\right) = \frac{\sin \pi a}{\pi a}. \tag{92}$$

A straightforward computation then yields

直接计算后可得

$$\det{}'\square = \left(2\tau_2 \eta\bar{\eta}\right)^2. \tag{93}$$

Putting everything together, one finds $L/\left(\sqrt{\tau_2}\eta\bar{\eta}\right)$, up to an overall constant.

合并所有结果后，在整体常数范围内我们得到 $L/\left(\sqrt{\tau_2}\eta\bar{\eta}\right)$ 。

The contribution of the reparametrisation ghosts to the vacuum energy proceeds in a similar fashion, although one has to properly treat the zero modes. From the Riemann-Roch theorem we know that there is one zero mode for each ghost and, therefore, one should compute

重参数化鬼对真空能的贡献也遵循类似的方式，但需要正确处理零模。根据黎曼-罗赫定理，我们知道每个鬼都对应一个零模，因此我们需要计算

$$\int [\mathcal{D}b]\,[\mathcal{D}c]\,\left[\mathcal{D}\bar{b}\right][\mathcal{D}\bar{c}]\, bc\bar{b}\bar{c} e^{-S_{\text{ghost}}}. \tag{94}$$

The ghost insertions simply soak up the zero modes from the integration measure, so that their contribution to the path integral no longer vanishes, and gives $\phi_{0,0}^4 = \tau_2^{-2}$. The non-zero modes instead contribute with

鬼插入项正好吸收了积分测度中的零模，因此它们对路径积分的贡献不再为零，结果为 $\phi_{0,0}^4 = \tau_2^{-2}$ 。而非零模的贡献为

$$\det{}'\nabla_z \det{}'\nabla_{\bar{z}} \sim \det{}'\square = \left(2\tau_2\eta\bar{\eta}\right)^2. \tag{95}$$

Assembling the contributions of the 26 bosonic coordinates X^{μ} , together with those of the ghosts, one recovers the result

汇总 26 个玻色坐标 X^μ 与鬼场的贡献后，我们可以重新得到结果

$$\log \mathcal{Z} = V_{26} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^{14}} \frac{1}{\eta^{24}\bar{\eta}^{24}}, \tag{96}$$

which precisely matches Eq. (34) for $D = 26$ upon restriction of the integral to the fundamental domain.

当把积分限制在基本域上时，该结果与 $D = 26$ 对应的式 (34) 完全一致

Fermionic Strings

费米子弦

In spite of successfully providing a quantum theory of gravity, the simple bosonic string model discussed so far clearly has a number of drawbacks. Its spectrum necessarily contains a tachyon, while spacetime fermions are absent. In order to overcome these problems, one is lead to introduce additional degrees of freedom on the string worldsheet [72]. The natural choice is two dimensional Majorana fermions ψ^μ , so that the Polyakov action, in a suitable generalisation of the conformal gauge, reads

尽管前文讨论的简单玻色弦模型成功给出了引力的量子理论，它显然存在不少缺陷: 其能谱必然包含快子，且不存在时空费米子。为了解决这些问题，人们需要在弦世界面引入额外自由度 [72]。最自然的选择是二维马约拉纳费米子 ψ^μ ，因此在共形规范的合适推广下，Polyakov 作用量可写为

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left(\partial_a X^\mu \partial^a X_\mu - i\bar{\psi}^\mu \rho^a \partial_a \psi_\mu \right), \tag{97}$$

where $\rho^0 = \sigma^2$ and $\rho^1 = i\sigma^1$ being the two-dimensional Dirac matrices satisfying the Clifford algebra $\{\rho^a, \rho^b\} = -2\eta^{ab}$ and the index $\mu = 0, \ldots, D-1$ runs over D -dimensional spacetime. In addition to global Poincaré symmetry and conformal invariance on the worldsheet, this action is also invariant under the supersymmetry transformation [73]

其中 $\rho^0 = \sigma^2$ 和 $\rho^1 = i\sigma^1$ 是满足克利福德代数 $\{\rho^a, \rho^b\} = -2\eta^{ab}$ 的二维狄拉克矩阵，指标 $\mu = 0, \ldots, D-1$ 遍历 D 维时空。除了整体庞加莱对称性和世界面上的共形不变性，该作用量还满足超对称变换下的不变性 [73]

$$\delta X^\mu = \bar{\varepsilon}\psi^\mu, \ \delta\psi^\mu = -i\rho^a \varepsilon \partial_a X^\mu, \tag{98}$$

with real parameter ε , transforming as a Majorana spinor. [1] The Dirac equation of motion implies that the two components of ψ^μ propagate independently as left or right movers, in accordance with the fact that, in two dimensions, irreducible spinorial representations of the Lorentz group are Majorana-Weyl fermions. As a consequence, the supersymmetry transformations independently rotate the left and right moving fields

其中实参数 ε 按马约拉纳旋量变换。[1] 运动方程狄拉克方程表明，ψ^μ 的两个分量分别以左行波和右行波独立传播，这符合二维洛伦兹群不可约旋量表示就是马约拉纳-外尔费米子的结论。因此，超对称变换可对左行和右行场分别独立旋转

$$\delta X^{\mu}_{\mathrm{L,R}}\left(\sigma^{\pm}\right) = \mp i\varepsilon_{\pm}\psi^{\mu}_{\pm},\ \delta\psi^{\mu}_{\pm}\left(\sigma^{\pm}\right) = \pm 2\varepsilon_{\pm}\partial_{\pm}X^{\mu}, \tag{99}$$

thus, respecting the factorisation of two-dimensional conformal symmetry. The energy-momentum tensor T and the supercurrent G read

因此满足二维共形对称性的因子分解性质。能量动量张量 T 和超流 G 可写为

$$T_{\pm\pm} = \partial_{\pm}X^{\mu}\partial_{\pm}X_{\mu} + \frac{i}{2}\psi^{\mu}_{\pm}\partial_{\pm}\psi_{\pm\mu},\ G_{\pm} = \psi^{\mu}_{\pm}\partial_{\pm}X_{\mu}. \tag{100}$$

The vanishing of energy-energy momentum tensor, together with the structure of the supersymmetry algebra $GG \sim T$, also requires the vanishing of the supercurrent. This constraint can be also seen to arise from the consistent coupling of the worldsheet fermions to two-dimensional supergravity.

能量动量张量为零，结合超对称代数 $GG \sim T$ 的结构，同样要求超流为零。也可以看出该约束来源于世界面费米子与二维超引力的一致耦合。

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[1] Actually, this transformation only realises a faithful representation of the supersymmetry algebra on-shell, where indeed the bosonic and fermionic degrees of freedom match. An off-shell realisation would require the introduction of the auxiliary real scalar field B^μ , entering the action via the bilinear $-B^\mu B_\mu$. The transformation of the fermion then involves the additional term $B^\mu\varepsilon$, while $\delta B^\mu = -i\bar{\varepsilon}\rho^a\partial_a\psi^\mu$.

[1] 实际上，该变换仅在壳实现了超对称代数的忠实表示，此时玻色子和费米子自由度恰好匹配。要得到脱壳实现则需要引入辅助实标量场 B^μ ，它通过双线性项 $-B^\mu B_\mu$ 进入作用量。费米子的变换会多出额外项 $B^\mu\varepsilon$ ，同时 $\delta B^\mu = -i\bar{\varepsilon}\rho^a\partial_a\psi^\mu$ 。

As usual, the equations of motion have to be supplemented by appropriate boundary conditions. For the bosonic coordinates they were already discussed in section ”The Bosonic String,” together with the corresponding mode expansions. In the case of closed strings, the vanishing of the boundary terms for the fermions simply implies periodicity or anti-periodicity in the σ variable. In the case of open strings, the boundary conditions imply $\psi^\mu_+\delta\psi_{+\mu} = \psi^\mu_-\delta\psi_{-\mu}$ at each endpoint; this leads to the two possibilities $\psi^\mu_+ = \pm\psi^\mu_-$ at $\sigma = 1$, upon conventionally fixing $\psi^\mu_+ = \psi^\mu_-$ at $\sigma = 0$.

和通常情况一样，运动方程需要补充合适的边界条件。玻色坐标的边界条件和对应的模展开已经在“玻色弦”一节讨论过了。对于闭弦，费米子边界项为零，这直接意味着坐标在 σ 变量下满足周期性或反周期性。对于开弦，边界条件要求两个端点都满足 $\psi^\mu_+\delta\psi_{+\mu} = \psi^\mu_-\delta\psi_{-\mu}$ ；在按惯例将 $\psi^\mu_+ = \psi^\mu_-$ 固定在 $\sigma = 1$ 处的 $\sigma = 0$ 后，就得到了两种可能情况 $\psi^\mu_+ = \pm\psi^\mu_-$ 。

In the closed string case, the mode expansions, thus, read

因此，闭弦的模展开可写为

$$\psi_{+}^{\mu}=\sqrt{2\pi\alpha'}\sum_{n\in\mathbb{Z}}d_{n}^{\mu}e^{-2\pi in\sigma^{+}},\ \psi_{+}^{\mu}=\sqrt{2\pi\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}b_{r}^{\mu}e^{-2\pi ir\sigma^{+}}, \tag{101}$$

for periodic and anti-periodic fermions, respectively. Similar expressions hold for the right movers ψ_{-}^{μ}, with $d\rightarrow\tilde{d}, b\rightarrow\tilde{b}$ and $\sigma^{+}\rightarrow\sigma^{-}$. In the open string case, we have

分别对应周期费米子和反周期费米子。右行波 ψ_{-}^{μ} 有类似的表达式，只需替换为 $d\rightarrow\tilde{d}, b\rightarrow\tilde{b}$ 和 $\sigma^{+}\rightarrow\sigma^{-}$。开弦的情况我们有

$$\psi_{\pm}^{\mu}=\sqrt{\pi\alpha'}\sum_{n\in\mathbb{Z}}d_{n}^{\mu}e^{-i\pi n\sigma^{\pm}},\ \psi_{\pm}^{\mu}=\sqrt{\pi\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}}b_{r}^{\mu}e^{-i\pi r\sigma^{\pm}} \tag{102}$$

for the two cases $\psi_{+}^{\mu}=\pm\psi_{-}^{\mu}$ at $\sigma=1$, respectively. For both closed and open strings, integer modes define the Ramond (R) sector [74,75], while the half-integer ones define the Neveu-Schwarz (NS) sector [76-78].

分别对应 $\sigma=1$ 处的两种情况 $\psi_{+}^{\mu}=\pm\psi_{-}^{\mu}$。对于闭弦和开弦，整数模定义了拉蒙德 (R) 扇区 [74,75]，半整数模定义了纳维-施瓦茨 (NS) 扇区 [76-78]。

The canonical quantisation of the fermions amounts to imposing the equal-time anti-commutation relations $\left\{\psi_{\pm}^{\mu}(\sigma),\psi_{\pm}^{\nu}(\sigma')\right\}=\pi\eta^{\mu\nu}\delta(\sigma-\sigma')$. Similarly to the bosonic string, the Fock space contains negative norm states, generated by the Fourier modes of ψ^{0} in both NS and R sectors. These are non-propagating degrees of freedom, which are removed by imposing that physical states lie in the cohomology of $G_{\pm}$, and it can be shown that this happens only in $D=10$ spacetime dimensions.

对费米子的正则量子化等价于施加等时反对易关系 $\left\{\psi_{\pm}^{\mu}(\sigma),\psi_{\pm}^{\nu}(\sigma')\right\}=\pi\eta^{\mu\nu}\delta(\sigma-\sigma')$。与玻色弦类似，福克空间存在由 ψ^{0} 的傅里叶模在 NS sector 和 R sector 中生成的负范态。这些是非传播自由度，可通过要求物理态处于 $G_{\pm}$ 的上同调中将其移除，且可证明该条件仅在 $D=10$ 时空维度下成立。

An alternative way to quantise the action (97) is to resort to light-cone quan-tisation. As in the bosonic case, there is an infinite number of conserved Noether charges associated to $T_{\pm\pm}$ and $G_{\pm}$. Indeed, any current of the form $f(\sigma^{\pm})T_{\pm\pm}$ and $g(\sigma^{\pm})G_{\pm}$ is trivially conserved for any f and g. This implies the existence of an infinite number of generators that extend the bosonic conformal symmetry to the infinite-dimensional superconformal group. Making use of these symmetries, we can set $X^{+}=x^{+}+2\pi\alpha' p^{+}\tau$ and $\psi^{+}=0$, so that the energy-momentum tensor and supercurrent constraints can be solved for

对作用量 (97) 的另一种量子化方法是光锥量子化。和玻色弦的情况一样，存在与 $T_{\pm\pm}$ 和 $G_{\pm}$ 相关的无穷多守恒诺特荷。事实上，对任意 f 和 g，任意形如 $f(\sigma^{\pm})T_{\pm\pm}$ 和 $g(\sigma^{\pm})G_{\pm}$ 的流都是平凡守恒的。这说明存在无穷多生成元，将玻色共形对称性拓展为无穷维超共形群。利用这些对称性，我们可以设定 $X^{+}=x^{+}+2\pi\alpha' p^{+}\tau$ 和 $\psi^{+}=0$，从而求解能量动量张量和超流约束

$$\partial_{\pm}X^{-}=\frac{1}{2\pi\alpha' p^{+}}\left(\partial_{\pm}X^{i}\partial_{\pm}X^{i}+\frac{i}{2}\psi_{\pm}^{i}\partial_{\pm}\psi_{\pm}^{i}\right),\ \psi_{\pm}^{-}=\frac{1}{\pi\alpha' p^{+}}\psi_{\pm}^{i}\partial_{\pm}X^{i}.$$

(103)

As a result, only the transverse (super)coordinates X^i, ψ^i are physical and generate a Fock space with positive norm. Indeed, plugging in the mode expansions, we find

最终，只有横向 (超) 坐标 X^i, ψ^i 是物理的，它们生成正范福克空间。代入模展开后，我们得到

$$\alpha_m^- = \frac{1}{\sqrt{2\alpha'} p^+}\left[\sum_{n\in\mathbb{Z}} \alpha_n^i \alpha_{m-n}^i + \sum_{r\in\mathbb{Z}+\frac{1}{2}} \left(\frac{m}{2} - r\right) b_r^i b_{m-r}^i\right] \tag{104}$$

and

和

$$b_r^- = \sqrt{\frac{2}{\alpha'}} \frac{1}{p^+} \sum_{s\in\mathbb{Z}+\frac{1}{2}} \alpha_{r-s}^i b_s^i, \tag{105}$$

for the NS sector of the left-movers of closed strings. Similar expressions can be derived for the right-movers, as well as for the R sectors. In the open string case one simply multiplies the r.h.s. of both equations by a factor of 1/2 .

适用于闭弦左行波的 NS sector。右行波以及 R sector 也可以推导出类似表达式。开弦情况下，我们只需在两个方程的右侧都乘以因子 1/2 。

As in the bosonic case, among the infinite relations (104), the $m = 0$ one plays a special role, since it provides the mass-shell condition. Recalling that $\alpha_0^\mu = \sqrt{2\alpha'} p^\mu$ for open strings, while $\alpha_0^\mu = \tilde{\alpha}_0^\mu = \sqrt{\alpha'/2} p^\mu$ for closed strings, one finds

和玻色弦的情况一样，在无穷多关系式 (104) 中，$m = 0$ 对应的关系式起着特殊作用，它给出了在壳质量条件。回顾开弦满足 $\alpha_0^\mu = \sqrt{2\alpha'} p^\mu$ ，闭弦满足 $\alpha_0^\mu = \tilde{\alpha}_0^\mu = \sqrt{\alpha'/2} p^\mu$ ，可得

$$M_{\text{open}}^2 = \frac{1}{\alpha'}\left(N_X + N_\psi - \frac{D-2}{16}\right), \tag{106}$$

in the NS sector, and

在 NS sector 中为,

$$M_{\text{open}}^2 = \frac{1}{\alpha'}\left(N_X + N_\psi\right), \tag{107}$$

in the R sector. In the closed string case, the left movers contribute with

在 R sector 中为。闭弦情形下，左行波的贡献为

$$M_{\text{closed},L}^2 = \frac{4}{\alpha'}\left(N_X + N_\psi - \frac{D-2}{16}\right), \tag{108}$$

in the NS sector, and

NS sector 中为,

$$M^2_{\text{closed },L} = \frac{4}{\alpha'}\left(N_X + N_\psi\right), \tag{109}$$

in the R sector, with similar expressions for the right moving mass. Closed string states are then built by combining the left and right-moving oscillators while respecting the level matching condition $M^2_{\text{closed },L} = M^2_{\text{closed },R}$. In these expressions, we used the definition of N_X given in section "The Bosonic String," while

R sector 中为，右行波质量也有类似表达式。闭弦态由左行振子和右行振子组合得到，同时满足能级匹配条件 $M^2_{\text{closed },L} = M^2_{\text{closed },R}$。这些表达式中，我们沿用了“玻色弦”章节给出的 N_X 定义，且

$$N_\psi = \begin{cases} \sum\limits_{r=1/2}^{\infty} r b^i_{-r} b^i_r & \text{in the NS sector,} \\ \sum\limits_{n=1}^{\infty} n d^i_{-n} d^i_n & \text{in the R sector.} \end{cases} \tag{110}$$

Furthermore, the zero point energy inside the brackets is computed via zeta function regularisation and each periodic boson contributes with -1/24, each R fermion contributes with +1/24, while each NS fermion contributes with -1/48 . The overall vanishing of the zero point energy in the R sector reflects the fact that the boundary conditions of bosons and fermions respect supersymmetry. In the NS sector, instead, the anti-periodicity of the fermions "spontaneously break" worldsheet supersymmetry and represent the simplest realisation of the Scherk-Schwarz mechanism [79].

此外, 括号内的零点能通过 ζ 函数正则化计算, 每个周期玻色子贡献-1/24, 每个 R 费米子贡献 +1/24, 每个 NS 费米子贡献-1/48。R sector 中零点能整体为零，反映了玻色子和费米子的边界条件满足超对称这一事实。而在 NS sector 中，费米子的反周期性“自发破缺”了世界面超对称，这是谢尔克-施瓦茨机制最简单的实现 [79]。

We now have all the ingredients to discuss the light spectrum, starting from the open strings. In the NS sector, the vacuum $|0\rangle$ is a spacetime scalar with mass $M^2 = -(D-2)/16\alpha'$ and is, thus, tachyonic (for $D > 2$). The first excited level is $b^i_{-1/2}|0\rangle$ transforming in the vectorial representation of the little group $\mathrm{SO}(D-2)$. This is clearly compatible with Lorentz invariance if and only if this state is massless, which is the case only in $D = 10$ dimensions. Indeed, it can be shown that the spacetime Lorentz algebra is properly realised in this critical dimension. Massive states can be similarly constructed by the (repeated) action of α^i_{-n} and b^i_{-r} and correspond to higher spin fields. In the R sector, instead, the vacuum is massless, but is no longer a singlet. This can be easily seen from the fact that d^i_0 commutes with the Hamiltonian and satisfies the Clifford algebra $\left\{d^i_0, d^j_0\right\} = 2\delta^{ij}$. As a result, the vacuum transforms in the spinorial representations $\mathbf{8}_s$ and $\mathbf{8}_c$ of the spacetime little group $\mathrm{SO}(8)$, corresponding to the Majorana-Weyl fermions of opposite chirality s and c . This implies that the superstring in the R sector describes spacetime fermions.

现在我们已经具备了所有讨论光频谱的条件，我们从开弦开始讨论。在 NS 扇区中，真空 $|0\rangle$ 是质量为 $M^2 = -(D-2)/16\alpha'$ 的时空标量，因此是快子 (对于 $D > 2$)。第一激发能级是 $b^i_{-1/2}|0\rangle$ ，按照小群 $SO(D-2)$ 的矢量表示变换。当且仅当该态为无质量态时，它才明显满足洛伦兹不变性，而只有在 $D = 10$ 维中才满足这一条件。事实上可以证明，时空洛伦兹代数在这个临界维数中可以正确实现。有质量态可以通过 α^i_{-n} 和 b^i_{-r} 的 (重复) 作用类似构造，对应更高自旋场。而在 R 扇区中，真空是无质量的，但不再是单态。这点可以从以下事实很容易看出: d^i_0 与哈密顿量对易，并且满足克利福德代数 $\left\{d^i_0, d^j_0\right\} = 2\delta^{ij}$ 。因此，真空按照时空小群 $SO(8)$ 的旋量表示 $\mathbf{8}_s$ 和 $\mathbf{8}_c$ 变换，对应相反手征性的马约拉纳-外尔费米子 s 和 c 。这说明 R 扇区的超弦描述时空费米子。

The closed string spectrum can be built by combining left and right movers and one has to distinguish between four possibilities. In the NS-NS sector is bosonic and its lightest state is the vacuum $|0\rangle_L \otimes |0\rangle_R$, which clearly satisfies level matching and describes a tachyonic scalar of mass $M^2 = -(D-2)/4\alpha'$. The next level-matched states originate from $b^i_{-1/2}\tilde{b}^j_{-1/2}$ acting on the vacuum. This decomposes into the symmetric traceless, the anti-symmetric and the singlet representations of $SO(D-2)$, and consistency with Lorentz symmetry requires that these states be massless, which again fixes $D = 10$. These states are identified as the graviton $G_{\mu\nu}$, the rank-2 anti-symmetric tensor $B_{\mu\nu}$, and the dilaton Φ . Also the R-R sector is bosonic, and its tower of states starts already at the massless level. To identify the spacetime representations of the latter states, we have to consider the tensor product decomposition of $(\mathbf{8}_s \oplus \mathbf{8}_c) \otimes (\mathbf{8}_s \oplus \mathbf{8}_c)$. As a result, $\mathbf{8}_s \otimes \mathbf{8}_s = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_+$ yields a scalar, a 2-form and a 4-form whose field strength is self-dual in $D = 10$ dimensions, $\mathbf{8}_c \otimes \mathbf{8}_c = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_-$ yields another scalar, another 2-form and a 4-form whose field strength is now anti self-dual, while $\mathbf{8}_s \otimes \mathbf{8}_c = \mathbf{8}_v \oplus \mathbf{56}$ yields a vector and a 3-form field, and similarly for $\mathbf{8}_c \otimes \mathbf{8}_s$. The NS-R sector is fermionic and its level-matched spectrum starts at the massless level. It is obtained by the tensor product decompositions $\mathbf{8}_v \otimes \mathbf{8}_s = \mathbf{56}_s + \mathbf{8}_c$ and $\mathbf{8}_v \otimes \mathbf{8}_c = \mathbf{56}_c + \mathbf{8}_s$. Therefore, it comprises two spin 3/2 fields of s and c chirality, known as the gravitini, and two spin 1/2 fields of c and s chirality, known as the dilatini. The R-NS sector gives an additional copy of the NS-R spectrum.

闭弦谱可以通过组合左行模和右行模构建，总共需要区分四种情形。NS-NS 扇区是玻色性的，其最轻的态是真空 $|0\rangle_L \otimes |0\rangle_R$ ，它显然满足能级匹配条件，描述一个质量为 $M^2 = -(D-2)/4\alpha'$ 的快子标量。下一个满足能级匹配的态由作用在真空上的 $b^i_{-1/2}\tilde{b}^j_{-1/2}$ 产生。这可以分解为 $SO(D-2)$ 的对称无迹表示、反对称表示和单态表示，洛伦兹对称性的自洽性要求这些态是无质量的，这再次确定了 $D = 10$ 。这些态分别对应引力子 $G_{\mu\nu}$ 、二阶反对称张量 $B_{\mu\nu}$ 和 dilation Φ 。R-R 扇区同样是玻色性的, 它的态谱从无质量能级就开始了。要确定这些态的时空表示, 我们需要考虑 $(\mathbf{8}_s \oplus \mathbf{8}_c) \otimes (\mathbf{8}_s \oplus \mathbf{8}_c)$ 的张量积分解。结果表明，$\mathbf{8}_s \otimes \mathbf{8}_s = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_+$ 给出一个标量、一个 2 形式和一个场强在 $D = 10$ 维中自对偶的 4 形式，$\mathbf{8}_c \otimes \mathbf{8}_c = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_-$ 给出另一个标量、另一个 2 形式和一个场强为反自对偶的 4 形式，$\mathbf{8}_s \otimes \mathbf{8}_c = \mathbf{8}_v \oplus \mathbf{56}$ 给出一个矢量场和一个 3 形式场，$\mathbf{8}_c \otimes \mathbf{8}_s$ 的情形同理。NS-R 扇区是费米性的，其满足能级匹配的谱从无质量能级开始，由张量积分解 $\mathbf{8}_v \otimes \mathbf{8}_s = \mathbf{56}_s + \mathbf{8}_c$ 和 $\mathbf{8}_v \otimes \mathbf{8}_c = \mathbf{56}_c + \mathbf{8}_s$ 得到。因此它包含两个分别具有 s 手征性和 c 手征性的自旋 3/2 场，称为引力微子；以及两个分别具有 c 手征性和 s 手征性的自旋 1/2 场，称为膨胀微子。R-NS 扇区给出 NS-R 谱的额外一份拷贝。

Actually, the closed string spectrum discussed so far yields an inconsistent string vacuum. One way to see this is the fact that keeping all the above states, together with their massive excitations, is incompatible with modular invariance. As we shall see, only certain sub-sectors will give a consistent string theory [80]. For open strings, the argument is slightly more involved and we will return to this in the next sections. In order to construct consistent vacua, it is useful to package the states of the NS and R sectors according to their worldsheet fermion parity $(-1)^{F_{\rm ws}}$. The latter is defined such that states built out of an even (resp. odd) number of b_r and/or d_n oscillators have $(-1)^{F_{\rm ws}} = +1$ (resp. -1). The parity of R vacua is positive for $|\mathbf{8}_s\rangle$ and negative for $|\mathbf{8}_c\rangle$ since, conventionally, $|8_s\rangle$ (resp. $|\mathbf{8}_c\rangle$) is built out of an even (resp. odd) number of d_0 oscillators. This packaging yields the four traces

实际上，目前讨论的闭弦能谱给出了一个不自洽的弦真空。一个佐证是: 保留上述所有态及其质量激发，与模不变性不兼容。我们将会看到，只有特定子扇区能给出自洽的弦论 [80]。对于开弦，这一论证稍复杂，我们会在后续章节再讨论。为构造自洽真空，我们可以方便地按世界面费米子宇称 $(-1)^{F_{\rm ws}}$ 对 NS 扇区和 R 扇区的态进行分类: 由偶数个 (奇数个) b_r 和/或 d_n 振子构造出的态，其 $(-1)^{F_{\rm ws}} = +1$ 为 1(-1)。按照约定，$|\mathbf{8}_s\rangle$ 的 R 真空宇称为正，$|\mathbf{8}_c\rangle$ 的 R 真空宇称为负，这是因为 $|8_s\rangle$ ($|\mathbf{8}_c\rangle$) 本身由偶数个 (奇数个) d_0 振子构造。这种分类给出四个迹

$$O_8(q) \equiv \mathrm{Tr}_{\rm NS}\left[\frac{1+(-1)^{F_{\rm ws}}}{2} q^{N_X+N_\psi-1/2}\right],$$

$$V_8(q) \equiv \mathrm{Tr}_{\rm NS}\left[\frac{1-(-1)^{F_{\rm ws}}}{2} q^{N_X+N_\psi-1/2}\right],$$

(111)

$$S_8(q) \equiv \mathrm{Tr}_{\rm R}\left[\frac{1+(-1)^{F_{\rm ws}}}{2} q^{N_X+N_\psi}\right],$$

$$C_8(q) \equiv \mathrm{Tr}_{\rm R}\left[\frac{1-(-1)^{F_{\rm ws}}}{2} q^{N_X+N_\psi}\right],$$

associated, respectively, to the singlet/adjoint, vectorial, and the two spinorial conjugacy classes of SO (8) . In general, these traces can actually be identified with the characters [81] of the level-one current algebra of SO $(2n)$, given by

分别对应 SO (8) 的单态/伴随表示、矢量表示以及两类旋量共轭类。一般来说，这些迹可对应 SO $(2n)$ 一级流代数的特征标 [81]，即

$$\begin{aligned}
O_{2n}(q) &= \frac{\vartheta_3(0\mid\tau)^n + \vartheta_4(0\mid\tau)^n}{2\eta^n(\tau)} \\
V_{2n}(q) &= \frac{\vartheta_3(0\mid\tau)^n - \vartheta_4(0\mid\tau)^n}{2\eta^n(\tau)}, \\
S_{2n}(q) &= \frac{\vartheta_2(0\mid\tau)^n + i^{-n}\vartheta_1(0\mid\tau)^n}{2\eta^n(\tau)}, \\
C_{2n}(q) &= \frac{\vartheta_2(0\mid\tau)^n - i^{-n}\vartheta_1(0\mid\tau)^n}{2\eta^n(\tau)}.
\end{aligned} \tag{112}$$

For a review on affine current algebras, see [82]. Here, we have introduced the four Jacobi theta functions

仿射流代数的综述可见 [82]。此处我们引入了四个雅可比 θ 函数

$$\vartheta_1(z \mid \tau) = 2\sin \pi z q^{1/8} \prod_{n>0} (1-q^n)\left(1-q^n e^{2\pi i z}\right)\left(1-q^n e^{-2\pi i z}\right),$$

$$\vartheta_2(z \mid \tau) = 2\cos \pi z q^{1/8} \prod_{n>0} (1-q^n)\left(1+q^n e^{2\pi i z}\right)\left(1+q^n e^{-2\pi i z}\right),$$

$$\vartheta_3(z \mid \tau) = \prod_{n>0} (1-q^n)\left(1+q^{n-1/2} e^{2\pi i z}\right)\left(1+q^{n-1/2} e^{-2\pi i z}\right), \tag{113}$$

$$\vartheta_4(z \mid \tau) = \prod_{n>0} (1-q^n)\left(1-q^{n-1/2} e^{2\pi i z}\right)\left(1-q^{n-1/2} e^{-2\pi i z}\right),$$

evaluated at $z = 0$. Notice that $\vartheta_1(0 \mid \tau)$ vanishes identically, reflecting the fact that at each mass level we have an equal number of fermionic states of opposite chirality. This degeneracy can be lifted by turning on suitable background magnetic fields, so that one may then really distinguish between the S_8 and C_8 traces. The generators T and S of the modular group $\mathrm{SL}(2;\mathbb{Z})$ have the well-defined action

在 $z = 0$ 处取值。注意 $\vartheta_1(0 \mid \tau)$ 恒为零，这反映出每个质量能级都存在相同数量手性相反的费米子态。开启合适的背景磁场可解除这种简并，从而真正区分 S_8 迹和 C_8 迹。模群 $\mathrm{SL}(2;\mathbb{Z})$ 的生成元 T 和 S 有明确定义的作用

$$T: \ \vartheta_1 \to e^{i\pi/4}\vartheta_1, \ \vartheta_2 \to e^{i\pi/4}\vartheta_2, \ \vartheta_{3,4} \to \vartheta_{4,3},$$

$$S: \ \vartheta_1 \to \sqrt{-i\tau}\vartheta_1, \ \vartheta_{2,4} \to \sqrt{-i\tau}\vartheta_{4,2}, \ \vartheta_3 \to \sqrt{-i\tau}\vartheta_3, \tag{114}$$

on the Jacobi theta functions with $z = 0$ which, together with the transformation properties (40) of the Dedekind eta function, implies the matrix representations

作用于带 $z = 0$ 的雅可比 θ 函数上，结合戴德金 η 函数的变换性质 (40)，可得到矩阵表示

$$T = e^{-i\pi n/12}\,\mathrm{diag}\left(1, -1, e^{i\pi n/4}, e^{i\pi n/4}\right), \ S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & i^{-n} & -i^{-n} \\ 1 & -1 & -i^{-n} & i^{-n} \end{pmatrix}.$$

(115)

on the space of the $\mathrm{SO}(2n)$ characters $\chi = \{O_{2n}, V_{2n}, S_{2n}, C_{2n}\}$.

作用于 $\mathrm{SO}(2n)$ 特征标空间 $\chi = \{O_{2n}, V_{2n}, S_{2n}, C_{2n}\}$ 上。

Consistent closed string vacua thus correspond to the sesquilinear combinations

因此自洽的闭弦真空对应如下半双线性组合

$$Z = \frac{1}{\eta^8 \overline{\eta}^8}\mathcal{T} \equiv \frac{1}{\eta^8 \overline{\eta}^8} \sum_{i,j} \overline{\chi}_i \mathcal{N}_{ij} \chi_j, \tag{116}$$

of left moving and right moving characters, with the matrix $\mathcal{N}_{ij}$ enforcing the Gliozzi-Scherk-Olive (GSO) projection [80]. Its entries can be either ± 1 or 0, and are required to satisfy spin-statistics and the (genus one) modular invariance constraints

由左行和右行特征标构成，其中矩阵 $\mathcal{N}_{ij}$ 用于实现格里奥齐-谢尔克-奥利维 (GSO) 投影 [80]。其矩阵元只能为 ± 1 或 0，且需要满足自旋统计定理和 (亏格 1 的) 模不变性约束

$$T^{\dagger}\mathcal{N}T=\mathcal{N},\ S^{\dagger}\mathcal{N}S=\mathcal{N}. \tag{117}$$

It can be shown that these conditions automatically guarantee higher genus modular invariance. There are only four inequivalent solutions,

可以证明，这些条件会自动保证高亏格模不变性。该约束仅有四个不等价解

$$\begin{gathered}
\mathcal{T}_{\mathrm{IIA}}=\left(V_8-S_8\right)\left(\bar{V}_8-\bar{C}_8\right) \\
\mathcal{T}_{\mathrm{IIB}}=\left|V_8-S_8\right|^2 \\
\mathcal{T}_{0\text{ A}}=\left|O_8\right|^2+\left|V_8\right|^2+S_8\bar{C}_8+C_8\bar{S}_8 \\
\mathcal{T}_{0B}=\left|O_8\right|^2+\left|V_8\right|^2+\left|S_8\right|^2+\left|C_8\right|^2,
\end{gathered} \tag{118}$$

corresponding to the type IIA, type IIB, type 0A and type 0B superstring theories in $D=10$ [80, 83, 84] .

分别对应 IIA 型、IIB 型、0A 型和 0B 型超弦理论，处于 $D=10$ [80, 83, 84] 。

The type IIA and type IIB superstrings enjoy $\mathcal{N}=(1,1)$ and $\mathcal{N}=(2,0)$ spacetime supersymmetry, respectively, and their corresponding partition functions vanish identically, as implied by the equatio identica satis abstrusa identity of Jacobi

IIA 型和 IIB 型超弦分别具有 $\mathcal{N}=(1,1)$ 和 $\mathcal{N}=(2,0)$ 时空超对称，由雅可比的恒等式可知它们的配分函数恒为零

$$\vartheta_3^4(0\mid\tau)-\vartheta_4^4(0\mid\tau)-\vartheta_2^4(0\mid\tau)=0. \tag{119}$$

Their massless excitations, thus, correspond to the field content of the associated supergravity multiplets. In type IIA they comprise $G_{\mu\nu}, B_{\mu\nu}$ and Φ from the NS-NS sector, a 1-form C_1 and a 3-form C_3 from the R-R sector, together with a pair of opposite-chirality gravitini and dilatini. In type IIB they comprise $G_{\mu\nu}, B_{\mu\nu}$ and Φ from the NS-NS sector, a 0-form C_0 , a 2-form C_2 and a 4-form (with self-dual field strength) $C_4^{(+)}$ from the R-R sector, together with two gravitini of the same (S) chirality and two dilatini of opposite (C) chirality. This chiral spectrum is actually anomaly-free [85], as guaranteed by modular invariance [86-88]. The type 0A and type 0B superstrings are non-supersymmetric and only contain bosonic excitations. The NS-NS sector is common to both theories and comprises a tachyon, $G_{\mu\nu}$, $B_{\mu\nu}$ and Φ . In the R-R sectors, type 0A comprises two 1-forms and two 3-forms from the R-R sectors, while type 0B comprises two 0-forms, two 2-forms, and one unconstrained 4-form.

因此，它们的无质量激发对应于相关超引力多重态的场内容。IIA 型中，无质量场包含 NS-NS 区的 $G_{\mu\upsilon}, B_{\mu\upsilon}$ 和 Φ ，R-R 区的 1-形式 C_1 和 3-形式 C_3 ，再加上一对手征性相反的引力微子和 dilatini(膨胀微子)。IIB 型中，无质量场包含 NS-NS 区的 $G_{\mu\upsilon}, B_{\mu\upsilon}$ 和 Φ ，R-R 区的 0-形式 C_0 、2-形式 C_2 和带自对偶场强的 4-形式 $C_4^{(+)}$ ，再加上两个相同 (S) 手征的引力微子和两个相反 (C) 手征的膨胀微子。这种手征谱实际上无反常 [85]，这由模不变性保证 [86-88]。0A 型和 0B 型超弦是非超对称的，仅包含玻色激发。两种理论的 NS-NS 区相同，包含一个快子、$G_{\mu\upsilon}$ 、$B_{\mu\upsilon}$ 和 Φ 。R-R 区中，0A 包含两个 1-形式和两个 3-形式，而 0B 包含两个 0-形式、两个 2-形式和一个无约束的 4-形式。

Although the partition functions in (118) define four independent string theories in ten dimensions there is, nevertheless, an interesting way to relate them. As an example, let us start from type IIB and observe that it is invariant under the action of the spacetime fermion parity $(-1)^F$. This discrete symmetry can then be gauged, by restricting the Hilbert space $\mathcal{H}_{\text{IIB}}$ to only those states invariant under this transformation. This amounts to computing

尽管 (118) 中的配分函数定义了十维中四个独立的弦论，但它们之间仍存在有趣的关联。举例来说，我们从 IIB 型出发，不难发现它在时空费米子宇称 $(-1)^F$ 的作用下保持不变。我们可以对这个离散对称性做规范变换: 将希尔伯特空间 $\mathcal{H}_{\text{IIB}}$ 限制为仅包含该变换下不变的态。这等价于计算

$$\mathcal{T}' = \text{Tr}_{\mathcal{H}_{\text{IIB}}}\left[\frac{1+(-1)^F}{2} q^{L_0}\bar{q}^{\bar{L}_0}\right] \tag{120}$$

$$= \frac{1}{2}|V_8 - S_8|^2 + \frac{1}{2}|V_8 + S_8|^2.$$

In accordance with the gauging of the spacetime fermion parity, the resulting spectrum only contains bosonic states from the NS-NS and R-R sectors of the type IIB superstring theory. However, $\mathcal{T}'$ is not modular invariant and, therefore, cannot describe a consistent string vacuum. A way out is to restore modular invariance with the addition of new ”twisted” states, which are not part of the original Hilbert space. In the case at hand, the only solution is

根据时空费米子宇称的规范变换结果，得到的谱仅包含 IIB 超弦理论 NS-NS 区和 R-R 区的玻色态。然而，$\mathcal{T}'$ 不具有模不变性，因此无法描述自洽的弦真空。解决方法是引入新的” 扭曲” 态来恢复模不变性，这些态不属于原本的希尔伯特空间。就当前情况而言，唯一解是

$$\mathcal{T}' \to \frac{1}{2}|V_8 - S_8|^2 + \frac{1}{2}|V_8 + S_8|^2 + \frac{1}{2}|O_8 - C_8|^2 + \frac{1}{2}|O_8 + C_8|^2, \tag{121}$$

which is nothing but the partition function of the type 0B theory. This construction [83, 84] is simplest instance of an orbifold, whereby discrete symmetries can be gauged at the cost of introducing new ”twisted” states which restore modular invariance. Similar constructions involving the full spacetime fermion parity or its right moving analogue $(-1)^{F_{\text{R}}}$, relate the four theories in (118). As we shall see in the next section, this procedure is an efficient way to construct new consistent closed string vacua.

它正是 0B 型理论的配分函数。这个构造 [83, 84] 是轨形最简单的例子: 对离散对称性做规范变换, 代价是引入新的” 扭曲” 态来恢复模不变性。类似构造涉及完整时空费米子宇称或其右动对应 $(-1)^{F_R}$ ，可以关联 (118) 中的四个理论。正如我们下一节将看到的，这个过程是构造新自洽闭弦真空的有效方法。

Heterotic Strings

杂化弦

In the previous section, we saw that an interesting way to generalise the bosonic string construction was to introduce additional fermionic degrees of freedom described in terms of Majorana-Weyl spinors $\psi^\mu_\pm$ carrying the same Lorentz index as the bosonic coordinates X^μ . This lead to a natural factorisation of the corresponding super-CFT into a left and right moving sector, where ψ^μ_+ and ψ^μ_- are independently rotated into $\partial_\pm X^\mu$ by supersymmetry transformations. This factorisation lies at the heart of string constructions and actually allows for an asymmetric generalisation. One may replace ψ^μ_- by N free fermions λ^A_- , which are now invariant under the spacetime Lorentz group but transform in the fundamental representation of SO (N) [89-91]. The resulting worldsheet action reads

在前一节中我们看到，推广玻色弦构造的一种有趣方式是引入额外的费米子自由度，这些自由度由马约拉纳-外尔旋量 $\psi^\mu_\pm$ 描述，该旋量与玻色坐标 X^μ 携带相同的洛伦兹指标。这自然将对应超共形场论分解为左行和右行两个扇区，其中超对称变换可将 ψ^μ_+ 和 ψ^μ_- 独立旋转为 $\partial_\pm X^\mu$ 。这种分解是弦构造的核心，实际上它允许进行不对称推广。我们可以将 ψ^μ_- 替换为 N 个自由费米子 λ^A_- ，这些费米子在时空洛伦兹群下不变，但在 SO (N) 的基础表示下变换 [89-91]，最终得到的世界面作用量为

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left(\partial_a X^\mu \partial^a X_\mu - 2i\psi^\mu_+\partial_-\psi_{+\mu} - 2i\lambda^A_-\partial_+\lambda^A_-\right), \tag{122}$$

and the quantisation proceeds as in the fermionic string with an important difference: the right-moving sector is no longer supersymmetric and, therefore, there is no associated conserved supercharge. One is, thus, left with the constraints

量子化过程与费米弦类似，但存在一个重要区别: 右行扇区不再具有超对称性，因此不存在关联的守恒超荷。我们最终得到的约束为

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu + \frac{i}{2}\psi^\mu_+\partial_+\psi_{+\mu} = 0,$$

$$T_{--} = \partial_- X^\mu \partial_- X_\mu + \frac{i}{2}\lambda^A_-\partial_-\lambda^A_- = 0, \tag{123}$$

$$G_+ = \psi^\mu_+\partial_+ X_\mu = 0.$$

As usual, in light-cone quantisation, the infinite conserved charges associated to T_{++} and T_{--} , allow us to eliminate the oscillators in $X^+ = x^+ + 2\pi\alpha' p^+\tau$. The chiral nature of worldsheet supersymmetry, however, now implies that the infinite conserved charges associated to G_+ only allow us to set $\psi_+^+ = 0$, while all right-moving λ_-^A oscillators must be retained. Taking this into account, the above constraints may be solved for X^- and ψ_+^- ,

和通常一样, 在光锥量子化中, 与 T_{++} 和 T_{--} 关联的无穷多守恒荷允许我们消除 $X^+ = x^+ + 2\pi\alpha' p^+\tau$ 中的振子。然而，世界面超对称的手征性现在意味着，与 G_+ 关联的无穷多守恒荷仅允许我们固定 $\psi_+^+ = 0$ ，而所有右行 λ_-^A 振子都必须保留。考虑到这一点，上述约束可对 X^- 和 ψ_+^- 求解，得到

$$\partial_+ X^- = \frac{1}{2\pi\alpha' p^+}\left(\partial_+ X^i \partial_+ X^i + \frac{i}{2}\psi_+^i \partial_+ \psi_+^i\right),$$

$$\partial_- X^- = \frac{1}{2\pi\alpha' p^+}\left(\partial_- X^i \partial_- X^i + \frac{i}{2}\lambda_-^A \partial_- \lambda_-^A\right), \tag{124}$$

$$\psi_+^- = \frac{1}{\pi\alpha' p^+}\psi_+^i \partial_+ X^i.$$

These relations are enough to remove the negative-norm states associated to X^0 and to the left-moving ψ_+^0 , and include the mass-shell conditions for the heterotic string.

这些关系足以消除与 X^0 和左行 ψ_+^0 关联的负范数态，同时包含杂化弦的在壳条件。

Using the mode expansions for the bosons (9) and the left-moving fermions (101), together with the mode expansions

利用玻色子 (9) 和左行费米子 (101) 的模展开，结合模展开

$$\lambda_-^A = \sqrt{2\pi\alpha'}\sum_{n\in\mathbb{Z}} \tilde{\lambda}_n^A e^{-2\pi i n\sigma^-}, \quad \lambda_-^A = \sqrt{2\pi\alpha'}\sum_{r\in\mathbb{Z}+\frac{1}{2}} \tilde{\lambda}_r^A e^{-2\pi i r\sigma^-}, \tag{125}$$

for the periodic and anti-periodic right-moving fermions λ^A , respectively, and assuming that all λ^A ’ s carry the same periodicity conditions, one finds

分别对周期和反周期的右行费米子 λ^A ，并假设所有 λ^A 都满足相同的周期性条件，可得

$$M_{\rm L}^2 = \begin{cases} \frac{4}{\alpha'}\left(N_X + N_\psi - \frac{D-2}{16}\right) & \text{in the NS sector,} \\ \frac{4}{\alpha'}\left(N_X + N_\psi\right) & \text{in the R sector,} \end{cases} \tag{126}$$

for the left-moving mass, and

左行质量为，右行质量为

$$M_{\rm R}^2 = \begin{cases} \frac{4}{\alpha'}\left(N_X + N_\lambda - \frac{2D+N-4}{48}\right) & \text{in the anti-periodic } \lambda \text{ sector ,} \\ \frac{4}{\alpha'}\left(N_X + N_\lambda - \frac{D-N-2}{24}\right) & \text{in the periodic } \lambda \text{ sector ,} \end{cases} \tag{127}$$

for the right-moving mass. Clearly, physical states require the level-matching condition $M_{\rm L}^2 = M_{\rm R}^2$. Notice that also in the right-moving periodic λ sector, the zero modes $\tilde{\lambda}_0^A$ commute with the Hamiltonian and satisfy the SO (N) Clifford algebra $\{\tilde\lambda_0^A,\tilde\lambda_0^B\}=2\delta^{AB}$. Therefore, the periodic λ vacuum transforms as a spinor of SO (N) . Compatibility of the light spectrum with the spacetime Lorentz symmetry, requires also in this case $D=10$ dimensions, while fixing $N=32$. As in the type IIA/IIB superstrings, in the left-moving sector, the GSO projection removes the NS tachyonic vacuum and the Ramond $\mathbf{8}_c$ vacuum together with their excitations. In the right-moving sector, the GSO projection removes instead the states belonging to the conjugacy classes of the fundamental and one of the spinorial representations of Spin(32). The modular invariant partition function, thus, reads

对右行质量。显然，物理态需要满足能级匹配条件 $M_\mathrm{L}^2=M_\mathrm{R}^2$ 。注意，即使在右行周期 λ 扇区，零模 $\tilde\lambda_0^A$ 也与哈密顿量对易，且满足 SO (N) 克利福德代数 $\{\tilde\lambda_0^A,\tilde\lambda_0^B\}=2\delta^{AB}$ 。因此，周期 λ 真空作为 SO (N) 的旋量变换。光锥谱与时空洛伦兹对称性的相容性要求在这种情况下也满足 $D=10$ 维，同时固定 $N=32$ 。与 IIA/IIB 型超弦类似，在左行扇区，GSO 投影会消除 NS 快子真空、朗道 $\mathbf{8}_c$ 真空以及它们的激发。在右行扇区，GSO 投影则会消除属于 Spin(32) 基础表示和其中一个旋量表示共轭类的态。因此，模不变配分函数为

$$\mathcal{T}_{\mathrm{SO}(32)}=(V_8-S_8)\left(\bar O_{32}+\bar S_{32}\right). \tag{128}$$

The massless states comprise the graviton, the B -field and the dilaton from $b^i_{-1/2}\tilde\alpha^j_{-1}\,|0\rangle_L\otimes|\,0\rangle_R$, the Rarita-Schwinger field of chirality s and a Majorana-Weyl spinor of chirality c from $\tilde\alpha^j_{-1}\,|\mathbf{8}_s\rangle_L\otimes|\,0\rangle_R$, together with 496 gauge bosons and s Majorana-Weyl fermions from $b^i_{-1/2}\tilde\lambda^A_{-1/2}\tilde\lambda^B_{-1/2}\,|0\rangle_L\otimes|\,0\rangle_R$ and $\tilde\lambda^A_{-1/2}\tilde\lambda^B_{-1/2}|\mathbf{8}_s\rangle_L\otimes|0\rangle_R$, respectively. This spectrum enjoys $\mathcal{N}=(1,0)$ supersymmetry in ten dimensions, and the aforementioned states form the gravity multiplet and the gauge multiplet of SO(32). This spectrum is free of irreducible gravitational and gauge anomalies [85], while the reducible anomalies are cancelled by the Green-Schwarz mechanism [92] (see [93,94] for a review of anomaly cancellation in string theory).

无质量态包含来自 $b^i_{-1/2}\tilde\alpha^j_{-1}\,|0\rangle_L\otimes|\,0\rangle_R$ 的引力子、B 场和伸缩子，来自 $\tilde\alpha^j_{-1}\,|\mathbf{8}_s\rangle_L\otimes|\,0\rangle_R$ 的手征为 s 的拉里塔-施温格场与手征为 c 的马约拉纳-魏尔旋量，再加上分别来自 $b^i_{-1/2}\tilde\lambda^A_{-1/2}\tilde\lambda^B_{-1/2}\,|0\rangle_L\otimes|\,0\rangle_R$ 和 $\tilde\lambda^A_{-1/2}\tilde\lambda^B_{-1/2}|\mathbf{8}_s\rangle_L\otimes\ |0\rangle_R$ 的 496 个规范玻色子与 s 个马约拉纳-魏尔费米子。该谱在十维中满足 $\mathcal{N}=(1,0)$ 超对称，上述态构成了 SO(32) 的引力多重态与规范多重态。该谱不存在不可约引力反常与规范反常 [85]，而可约反常可通过格林-施瓦茨机制消去 [92](弦论中的反常消去综述见 [93,94])。

In the above, we imposed the same periodicity conditions for all 32 λ's, which gave rise to the SO(32) heterotic string with $\mathcal{N}=(1,0)$ spacetime supersymmetry. This is not the only allowed choice, and one could instead split the right-moving fermions into sets obeying different periodicity conditions. If one insists on preserving spacetime supersymmetry in $D=10$, the GSO projection implies that the partition function factorises into the holomorphic contribution V_8-S_8 times an anti-holomorphic one associated to the gauge degrees of freedom. As a result, the latter must be modular invariant by itself and, it turns out, that there are exactly two choices involving 32 fermions: the $\bar O_{32}+\bar S_{32}$ combination discussed above, and $\left(\bar O_{16}+\bar S_{16}\right)\left(\bar O_{16}+\bar S_{16}\right)$ which gives rise to the celebrated $\mathrm{E}_8\times\mathrm{E}_8$ heterotic string with partition function

上文我们对全部 32 个 λ 施加了相同的周期性条件, 由此得到了具有 $\mathcal{N}=(1,0)$ 时空超对称的 SO(32) 杂化弦。这并非唯一允许的选择, 我们也可以将右行费米子拆分到满足不同周期性条件的集合中。如果要求保留 $D=10$ 中的时空超对称，GSO 投影表明配分函数可分解为全纯贡献 V_8-S_8 乘以与规范自由度相关的反全纯贡献。因此反全纯部分自身必须具备模不变性, 且可以证明, 对于 32 个费米子恰好存在两种选择: 上文讨论的 $\bar{O}_{32}+\bar{S}_{32}$ 组合, 以及著名的 $\mathrm{E}_8\times\mathrm{E}_8$ 杂化弦对应的 $\left(\bar{O}_{16}+\bar{S}_{16}\right)\left(\bar{O}_{16}+\bar{S}_{16}\right)$，其配分函数为

$$\mathcal{T}_{\mathrm{E}_8\times\mathrm{E}_8}=\left(V_8-S_8\right)\left(\bar{O}_{16}+\bar{S}_{16}\right)\left(\bar{O}_{16}+\bar{S}_{16}\right). \tag{129}$$

Aside from the $\mathcal{N}=(1,0)$ gravitational multiplet, the light spectrum comprises a vector multiplet transforming under the 248-dimensional adjoint representation of $\mathrm{E}_8\times\mathrm{E}_8$. Indeed, an E_8 gauge group is isomorphic to $\mathrm{Spin}\,(16)\,/\mathbb{Z}_2$, where one retains the conjugacy classes associated to the adjoint and one spinorial representation of Spin(16). Notice that we have exactly $248+248=496$ copies of a vector multiplet, as in the case of the SO(32) heterotic string, which is essential for gravitational anomaly cancellation.

除 $\mathcal{N}=(1,0)$ 引力多重态外，轻谱还包含一个在 $\mathrm{E}_8\times\mathrm{E}_8$ 的 248 维伴随表示下变换的矢量多重态。确实，E_8 规范群同构于 $\mathrm{Spin}\,(16)\,/\mathbb{Z}_2$ ，此时我们保留与 Spin(16) 的伴随表示和一个旋量表示相关的共轭类。注意我们恰好有 $248+248=496$ 个矢量多重态拷贝，和 SO(32) 杂化弦的情况一致，这对消去引力反常至关重要。

It is a property of two-dimensional CFTs that a pair of real free fermions $\lambda^{1,2}$ can be bosonised [95, 96] into a chiral compact scalar Φ at radius $\sqrt{\alpha'/2}$, known as the fermionic point. Indeed, the dictionary

二维共形场论有一个性质: 一对实自由费米子 $\lambda^{1,2}$ 可以玻色化 [95, 96] 为半径 $\sqrt{\alpha'/2}$ 处的手征紧致标量 Φ，该点称为费米点。对应关系如下

$$\lambda^{\pm}\equiv\frac{\lambda^2\pm i\lambda^1}{\sqrt{2}}\rightarrow e^{\pm i\sqrt{2/\alpha'}\Phi},\ \lambda^+\lambda^-\rightarrow i\bar{\partial}\Phi, \tag{130}$$

consistently reproduces the OPE relations. As a result, one may give an alternative description of the heterotic strings whereby the right-moving sector involves 16 compact scalars Φ^a . The chiral factorisation induced by the requirement of spacetime supersymmetry, together with the modular invariance of the holomorphic and anti-holomorphic sectors, implies that the compact Φ^a ’ s are associated to a 16-dimensional even, self-dual chiral lattice. The only two such cases are the root lattices of $\mathrm{Spin}\,(32)\,/\mathbb{Z}_2$ and $\mathrm{E}_8\times\mathrm{E}_8$, corresponding to the partition functions (128) and (129), respectively [89-91].

一致地重现了算符乘积展开关系。由此我们可以对杂化弦给出另一种描述: 其右行 Sector 包含 16 个紧化标量 Φ^a 。时空超对称要求诱导出的手征分解，结合全纯与反全纯 Sector 的模不变性，意味着这些紧化的 Φ^a 对应一个 16 维偶自对偶手征格。仅存在两种符合条件的情况，即根格 $\mathrm{Spin}\,(32)\,/\mathbb{Z}_2$ 和根格 $\mathrm{E}_8\times\mathrm{E}_8$ ，分别对应配分函数 (128) 和 (129) [89-91]。

Although these cases are the only possible choices of heterotic theories in ten dimensions enjoying spacetime supersymmetry, they do not exhaust the space of all consistent, modular invariant, heterotic vacua. The most notable example is the non-supersymmetric $\mathrm{SO}\,(16)\times\mathrm{SO}\,(16)$ theory of [83, 84, 97] . In this case, the modular invariant partition function

尽管这两种是十维时空超对称杂化弦理论仅有的可能选择，但它们并未穷尽所有自洽、模不变的杂化真空。最典型的例子就是 [83, 84, 97] 提出的非超对称 SO (16) × SO (16) 理论，其模不变配分函数

(131)

$$\mathcal{T}_{\mathrm{SO}(16)\times\mathrm{SO}(16)} = V_8\left(\bar{O}_{16}\bar{O}_{16} + \bar{S}_{16}\bar{S}_{16}\right) - S_8\left(\bar{O}_{16}\bar{S}_{16} + \bar{S}_{16}\bar{O}_{16}\right)$$

$$+O_8\left(\bar{V}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{V}_{16}\right) - C_8\left(\bar{V}_{16}\bar{V}_{16} + \bar{C}_{16}\bar{C}_{16}\right)$$

no longer factorises, and the light spectrum comprises the universal graviton, B - field and dilaton, gauge bosons in the adjoint representation of SO(16) × SO(16), left-handed fermions in the $(\mathbf{128},\mathbf{1})+(\mathbf{1},\mathbf{128})$ and right-handed fermions in the (16,16) representations. The would-be tachyonic state originating from O_8 is actually massive, because of level-matching, which makes this non-supersymmetric closed string theory unique in $D=10$.

不再满足分解性，轻谱包含通用的引力子、B 场和 dilaton (伸缩子)、SO(16) × SO(16) 伴随表示的规范玻色子、$(\mathbf{128},\mathbf{1})+(\mathbf{1},\mathbf{128})$ 表示的左手征费米子，以及 (16,16) 表示的右手征费米子。源自 O_8 的类快子态实际上因能级匹配获得质量，这使得该非超对称闭弦理论在 $D=10$ 中是独一无二的。

As in the superstring case, the SO (16) × SO (16) theory can be constructed from the supersymmetric $\mathrm{E}_8\times\mathrm{E}_8$ one by employing the $\mathbb{Z}_2$ orbifold, generated by $(-1)^{F+F_1+F_2}$. Here F is the usual spacetime fermion number, while $F_{1,2}$ are the analogous "fermion numbers" for each E_8 factor. Also in this case, modular invariance requires the presence of a twisted sector respecting the orbifold symmetry, which is precisely encoded in the second line of (131).

和超弦情况类似, SO (16)×SO (16) 理论可以通过利用 $(-1)^{F+F_1+F_2}$ 生成的 $\mathbb{Z}_2$ 轨形, 从超对称 $\mathrm{E}_8\times\mathrm{E}_8$ 理论构造得到。此处 F 是常规时空费米子数，$F_{1,2}$ 则是每个 E_8 因子对应的类似“费米子数”。该情形下，模不变性要求存在满足轨形对称性的扭曲 Sector，这一点正好由 (131) 的第二行编码。

The only other choices are non-supersymmetric and have gauge groups SO(32), SO (16)×E_8, $(\mathrm{SU}(2)\times\mathrm{E}_7)^2$, SO (8)× SO (24) , U (16) [83, 84] and E_8 [98] . However, all of them involve tachyonic states, which render them classically unstable.

仅有的其他可能都是非超对称的，其规范群为 SO(32)、SO (16) × E_8, $(\mathrm{SU}(2)\times\mathrm{E}_7)^2$, SO (8) × SO (24) , U (16) [83, 84] 和 E_8 [98] 。但所有这些理论都存在快子态，导致它们在经典层面不稳定。

Among these theories, the E_8 one is special, in that it has reduced rank and involves a current algebra of level 2. It can be constructed as a permutation orbifold, where the exchange of the two E_8 ’ s is accompanied by $(-1)^F$. The partition function reads

在这些理论中，E_8 理论十分特殊: 它的秩更低，且包含 2 级流代数。它可以构造为置换轨形，其中交换两个 E_8 的同时伴随 $(-1)^F$ 。其配分函数为

$$\mathcal{T}_{\mathrm{E}_8} = \frac{1}{2}\Big[(V_8-S_8)\,\overline{\chi}_8(\bar{q})\,\overline{\chi}_8(\bar{q}) + (V_8+S_8)\,\overline{\chi}_8(\bar{q}^2) \tag{132}$$

$$+\,(O_8-C_8)\,\overline{\chi}_8\left(\sqrt{\bar{q}}\right) + (O_8+C_8)\,\overline{\chi}_8\left(-\sqrt{\bar{q}}\right)\Big]\,,$$

where, for convenience, we denote by $\overline{\chi}_8 = \bar{O}_{16} + \bar{S}_{16}$ the chiral E_8 character and we have also introduced the hatted characters

为方便起见，我们用 $\overline{\chi}_8 = \bar{O}_{16} + \bar{S}_{16}$ 表示手征 E_8 特征标，同时引入了带帽特征标

$$\hat{\chi}(-\sqrt{q}) = e^{-i\pi(h-c/24)} \chi(-\sqrt{q}), \tag{133}$$

of conformal weight h and associated central charge c . The light spectrum comprises the universal graviton, B -field and dilaton, a vector and a Majorana fermion in the adjoint representation of E_8 , together with a singlet tachyon. This vacuum is a prototype example of a larger class of lower-dimensional constructions with reduced rank involving (freely-acting) permutation orbifolds, known as CHL strings [99].

共形权重为 h ，其关联中心荷为 c 。轻谱包括普适引力子、B 场与涨子，伴随表示为 E_8 的一个矢量和一个马约拉纳费米子，以及一个快子单态。该真空是一类规模更大、降维且约化秩的构造的原型范例，这类构造包含 (自由作用) 置换轨形，通称为 CHL 弦 [99]。

Open Strings and D-Branes

开弦与 D 膜

Until now, we have mainly focused on the construction of closed string vacua. As we shall see, open strings also lead to interesting, although less straightforward, constructions. Closed strings naturally require periodicity conditions for the X^μ coordinates, which imply that their centre of mass is free to move in the whole ten-dimensional spacetime. Open superstrings, instead, require the specification of Neumann or Dirichlet boundary conditions at the two endpoints, and this affects their propagation. The main difference between NN and DD strings lies in the zero modes of their X^μ coordinates, whereby only the former admit a centre of mass momentum, while the latter are stuck, as can be seen from Eqs. (10) and (11). Therefore, open strings with $p + 1$NN and $9 - p$DD boundary conditions specify a $(p + 1)$ -dimensional hypersurface along which the open strings are free to move, and naturally break $\mathrm{SO}(1, 9)$ down to $\mathrm{SO}(1, p) \times \mathrm{SO}(9 - p)$. In this way, the Lorentz index μ splits into $\mu \to (a, i)$, where $a = 0, \ldots, p$ spans the directions along the Dp brane, while $i = p + 1, \ldots, 9$ labels the transverse coordinates. This hypersurface is known as a Dp brane [100-102] (for reviews, see [33-35, 41, 44]) and fully specifies the set of boundary conditions of open strings. Following similar steps as in section ”Fermionic Strings,” light-cone quantisation yields the mass formula

迄今为止，我们主要聚焦于闭弦真空的构造。我们将会看到，开弦也能给出有趣的构造，只是过程不如闭弦直接。闭弦对 X^μ 坐标自然要求周期性条件，这意味着其质心可在整个十维时空内自由运动。而开超弦需要在两个端点处指定诺依曼边界条件或狄利克雷边界条件，这会影响开弦的传播。NN 弦与 DD 弦的主要区别在于 X^μ 坐标的零模: 只有 NN 弦存在质心动量，DD 弦则被束缚，这可以从式 (10) 和 (11) 看出。因此，带有 $p + 1$NN 和 $9 - p$DD 边界条件的开弦确定了一个 $(p + 1)$ 维超曲面，开弦可沿该超曲面自由运动，并且自然将 $\mathrm{SO}(1, 9)$ 破缺为 $\mathrm{SO}(1, p) \times \mathrm{SO}(9 - p)$ 。由此，洛伦兹指标 μ 拆分为 $\mu \to (a, i)$ ，其中 $a = 0, \ldots, p$ 张成沿 Dp 膜的方向，$i = p + 1, \ldots, 9$ 标记横向坐标。这个超曲面就是所谓的 Dp 膜 [100-102]，综述参见 [33-35, 41, 44]，它完全确定了开弦的所有边界条件。遵循与“费米弦”一节类似的步骤，光锥量子化给出质量公式

$$M^2_{\text{open}} = \frac{1}{\alpha'}\left(N_X + N_\psi + \Delta\right), \tag{134}$$

for open strings whose endpoints live on the same Dp brane, with Δ being the zero point energy which vanishes in the R sector, whereas it equals $-1/2$ in the NS sector. In the NS sector, the light spectrum contains the tachyonic vacuum $|0\rangle$, together | with a massless vector in $(p+1)$ dimensions from $b^a_{-1/2}|0\rangle$ and $9-p$ scalars from $b^i_{-1/2}|0\rangle$. The latter are actually associated to the position of the D-brane along the transverse directions. In the R sector, the vacuum is massless and, as usual, describes the $\mathbf{8}_s$ and $\mathbf{8}_c$ spinors of the original Lorentz group $\mathrm{SO}(1,9)$, which are then to be properly decomposed. All these fields are free to propagate only along the world-volume of the Dp brane.

端点位于同一 Dp 膜上的开弦，其中 Δ 是零点能，零点能在 R 区为零，在 NS 区等于 $-1/2$ 。在 NS 区，轻谱包含快子真空 $|0\rangle$, together | ，还有来自 $b^a_{-1/2}|0\rangle$ 的 $(p+1)$ 维无质量矢量，以及来自 $b^i_{-1/2}|0\rangle$ 的 $9-p$ 个标量。这些标量实际上对应 D 膜沿横向方向的位置。在 R 区，真空是无质量的，和通常情况一样，描述原洛伦兹群 $\mathrm{SO}(1,9)$ 的 $\mathbf{8}_s$ 和 $\mathbf{8}_c$ 旋量，之后需要对其进行适当分解。所有这些场都只能沿 Dp 膜的世界体积自由传播。

Also in this case, one has to properly truncate the Hilbert space in order to construct a consistent open string theory. The way to proceed, however, is drastically different from the closed string case, since the one-loop vacuum diagram associated to an open string does not enjoy modular invariance. Indeed, it is given by a Riemann surface with the topology of an annulus, where the two boundaries are traced by the string endpoints, as shown in Fig. 1. Such a Riemann surface has vanishing Euler characteristic and may be built from a double-covering torus via the anti-holomorphic involution $z \to 2-\bar{z}$. The compatibility of this involution with the torus identification $z \sim z + m\tau + 2n$, where $m, n \in \mathbb{Z}$, implies that the modulus of the double-covering torus is purely imaginary, a requirement clearly incompatible with modular invariance. Actually, the fact that the complex structure must be purely imaginary is not surprising since, in the case of open strings, one does not need to impose level-matching and, therefore, the Schwinger proper time τ_2 is sufficient to parametrise the vacuum diagram.

这种情况同样需要对希尔伯特空间做适当截断，才能构造自洽的开弦理论。但具体做法和闭弦情形截然不同，因为开弦对应的单圈真空图不具备模不变性。该图的拓扑为环形黎曼曲面，两个边界由弦端点扫出，如图 1 所示。这类黎曼曲面的欧拉示性数为零，可以通过反全纯对合 $z \to 2-\bar{z}$ 由双叶覆盖圆环构造得到。该对合与圆环标识 $z \sim z + m\tau + 2n$ (满足 $m, n \in \mathbb{Z}$) 的相容性要求，双叶覆盖圆环的模是纯虚数，这一要求显然与模不变性不兼容。实际上，复结构必须为纯虚数并不意外，因为对开弦而言不需要满足能级匹配，因此只需施温格固有时 τ_2 就可以参数化真空图。

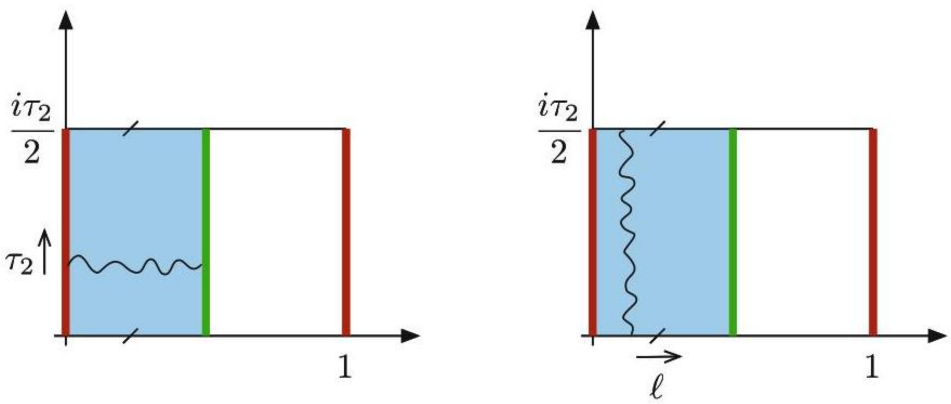

Fig. 1 The left figure shows the one-loop vacuum diagram of an open string which propagates for a (vertical) proper time τ_2 ; its end-points trace the two boundaries of an annulus. The right figure illustrates the tree-level propagation of a closed string bouncing between the two boundaries; the proper time ℓ now flows horizontally. Both figures also display the double covering torus with modulus $i\tau_2/2$

图 1 左图展示了开弦的单圈真空图，该开弦沿 (竖直方向) 固有时 τ_2 传播；其端点描出圆环的两条边界。右图展示了闭弦在两条边界之间反弹的树级传播；此时固有时 ℓ 沿水平方向流动。两图还展示了模为 $i\tau_2/2$ 的双覆盖环面

Another important difference is that, although the torus always describes the one-loop propagation of closed strings, independently of the choice of elementary cell, in open strings an inversion of τ_2 implies that time now flows horizontally as shown in Fig. 1 and calls for a completely different interpretation in terms of closed strings freely propagating between the two boundaries of a cylinder [103-106]. The main lesson to be drawn from this observation is that open strings alone do not define a unitary theory, since their endpoints may join to form a closed string and/or the loop diagrams admit dual descriptions in terms of open/closed propagation. Given this fact, the open string spectrum encoded in the annulus partition function must be compatible with the closed string spectrum propagating along the dual cylinder. It is this constraint that essentially replaces modular invariance and selects the correct GSO projection of the open string Hilbert space. Therefore, if we wish to consistently describe open strings together with type II (closed) superstrings, we must employ the same supersymmetric GSO projection [80], so that, aside from an irrelevant multiplicative volume factor, the annulus partition function reads

另一个重要区别是，尽管环面始终描述闭弦的单圈传播，且与基元胞的选择无关，但对开弦而言，τ_2 的反演意味着时间现在沿水平方向流动 (如图 1 所示)，需要用闭弦在圆柱两条边界之间自由传播来给出完全不同的诠释 [103-106]。从该观察可以得到的核心结论是，仅开弦本身无法定义幺正理论，因为开弦的端点可以接合形成闭弦，且/或圈图可以用开弦/闭弦传播给出对偶描述。因此，环形配分函数中编码的开弦谱必须与沿对偶圆柱传播的闭弦谱相容。正是这一约束取代了模不变性，选出了开弦希尔伯特空间正确的 GSO 投影。因此，如果我们想要一致地描述开弦与 II 型 (闭) 超弦，我们必须采用相同的超对称 GSO 投影 [80]，因此除了一个无关的整体体积因子外，环形配分函数可写为

$$\mathcal{A} = \int_0^\infty \frac{d\tau_2}{\tau_2} \frac{1}{\tau_2^{(p+1)/2} \eta^8\left(\frac{i\tau_2}{2}\right)} (V_8 - S_8)\left(\frac{i\tau_2}{2}\right), \tag{135}$$

where Dedekind and Jacobi functions naturally depend on the modulus $i\tau_2/2$ of the double-covering torus. Here, we have assumed that open strings have NN boundary conditions along $p+1$ coordinates, so that the massless spectrum corresponds to the dimensional reduction of a ten-dimensional vector and left-handed spinor on the $(p+1)$ -dimensional world-volume of the Dp brane. This is the field content of a vector supermultiplet in a theory with 16 supercharges.

其中戴德金函数与雅可比函数自然依赖于双覆盖环面的模 $i\tau_2/2$ 。这里我们假设开弦沿 $p+1$ 坐标满足 NN 边界条件，因此零质量谱对应十维矢量和左手旋诺在 Dp 膜的 $(p+1)$ 维世界体积上的维约化。这就是拥有 16 个超荷的理论中矢量超多重态的场内容。

The representation (135) of the open string vacuum amplitude is normally referred to as the loop or direct channel amplitude. The transformation $\tau_2 \rightarrow \ell = 2/\tau_2$ defines what is called the tree-level or transverse channel amplitude

开弦真空振幅的表示式 (135) 通常被称为圈道或直接道振幅。变换 $\tau_2 \rightarrow \ell = 2/\tau_2$ 定义了所谓的树道或横道振幅

$$\widetilde{\mathcal{A}} = 2^{-(p+1)/2} \int_0^\infty \frac{d\ell}{\ell^{(p-9)/2}} \frac{1}{\eta^8 (i\ell)} (V_8 - S_8)(i\ell), \tag{136}$$

which describes the propagation of NS-NS and R-R states for proper time ℓ between the two boundaries, i.e., the Dp branes. In the limit of an infinitely long cylinder, $\ell \rightarrow \infty$, only massless excitations survive and the resulting diagram involves 1-point functions coupling the massless fields to the boundaries, and an on-shell propagator evaluated at zero momentum. This shows that Dp -branes are physical objects carrying tension as well as charge for the R-R potentials [107], which identifies them as the BPS solitons of type II supergravity [108-110] (see also [111] for a review), preserving 16 of the original 32 supercharges. Since a $(p+1)$ -form potential naturally couples to a p -dimensional (static) source, Dp branes exist in type IIA (IIB) superstring theory for p even (odd).

它描述了 NS-NS 态和 R-R 态在两条边界即 Dp 膜之间沿固有时 ℓ 的传播。在无限长圆柱极限 $\ell \rightarrow \infty$ 下，只有零质量激发存活，得到的图包含将零质量场耦合到边界的一点函数，以及零动量处的在壳传播子。这表明 Dp 膜是携带张力和 R-R 势荷的物理客体 [107]，这说明它们是 II 型超引力的 BPS 孤子 [108-110](综述参见 [111])，保留了原 32 个超荷中的 16 个。由于 $(p+1)$ 形式势自然耦合到 p 维 (静态) 源，因此对于 p 为偶 (奇) 的情况，Dp 膜存在于 IIA(IIB) 超弦理论中。

Notice that, if the space transverse to the D-brane is compact, the theory of closed and open oriented strings discussed so far cannot yield a consistent vacuum. This is because, on a compact space, Gauss' law requires a neutral configuration of charges, so that Faraday lines emitted from positively charged sources are absorbed by negatively charged ones. A consistent vacuum configuration may still be built, but it requires the inclusion of unoriented strings, and we defer the relevant discussion to the next section.

需要注意，如果垂直于 D 膜的空间是紧致的，那么目前讨论的定向闭弦与开弦理论无法得到自洽的真空。这是因为在紧致空间上，高斯定律要求电荷呈中性构型，因此正电荷源发出的法拉第力线会被负电荷源吸收。我们仍然可以构造出自洽的真空构型，但这需要引入非定向弦，相关讨论我们留到下一节。

Before we conclude this section, it is instructive to consider the situation depicted in Fig. 2, where two parallel D-branes are separated by a distance δ . Here we can identify two types of open strings: those which start and end on the same D-brane and those which stretch between the two different D-branes. The former case is similar to what has been discussed so far, and the light spectrum contains a pair massless Abelian vectors $A_\mu^{1,2}$, each living on the world-volume of a D-brane. In the latter case, the mass operator is shifted by the distance δ ,

在结束本节之前，我们不妨来看图 2 描绘的情况: 两张平行 D 膜相距 δ 。这里我们可以区分两类开弦: 起止都在同一张 D 膜上的开弦，以及拉伸在两张不同 D 膜之间的开弦。前者和我们之前讨论的情况类似，其轻谱包含一对零质量阿贝尔矢量 $A_\mu^{1,2}$ ，每个矢量分别位于一张 D 膜的世界体积上。对于后者，质量算符会偏移距离 δ ,

$$M_{\text{open}}^2 = \frac{1}{\alpha'}\left(N_X + N_\psi + \Delta\right) + \frac{\delta^2}{\left(2\pi\alpha'\right)^2}, \tag{137}$$

so that the stretched strings produce a pair of massive vectors $A_\mu^\pm$, charged with respect to $A_\mu^{1,2}$. In the limit where $\delta \rightarrow 0$ and the D-branes form a stack, the $A_\mu^\pm$ become massless and it can be shown that the original gauge symmetry $\mathrm{U}(1)^2$ is enhanced to $\mathrm{U}(2)$ [112, 113] . This exercise can be repeated in the case where N D-branes are involved. When they form a single stack, the gauge symmetry is maximal, while the various separations induce the breaking $\mathrm{U}(N) \rightarrow \mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \ldots \times \mathrm{U}(N_m)$, with $N_1 + N_2 + \ldots + N_m = N$. This way of describing non-Abelian gauge symmetries in open strings is equivalent to the introduction of Chan-Paton factors λ_{ij}^a dressing string states. It has been shown [114,115] that the proper factorisation of scattering amplitudes restricts the consistent gauge group factors that can appear in open string constructions to $\mathrm{U}(N)$, $\mathrm{SO}(N)$ and $\mathrm{USp}(2N)$, with the last two requiring unoriented strings. Note that one may alternatively recover these gauge groups from the dynamics of additional degrees of freedom living at the two endpoints of an open string [116].

因此拉伸弦会产生一对带荷的大质量矢量 $A_\mu^\pm$ ，其荷对应于 $A_\mu^{1,2}$ 。在 $\delta \rightarrow 0$ 极限下，D 膜会形成一个堆，$A_\mu^\pm$ 变为无质量，可证明原规范对称性 $\mathrm{U}(1)^2$ 增强为 $\mathrm{U}(2)$ [112, 113] 。这套分析可以推广到涉及 N 个 D 膜的情况。当所有 D 膜形成单个堆时，规范对称性达到最大，而不同 D 膜之间的间隔会引发对称性破缺 $\mathrm{U}(N) \rightarrow \mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \ldots \times \mathrm{U}(N_m)$ ，得到 $N_1 + N_2 + \ldots + N_m = N$ 。这种在开弦中描述非阿贝尔规范对称性的方法，等价于为弦态引入陈-帕顿因子 λ_{ij}^a 。已有研究 [114,115] 证明，散射振幅的正确因式化会限制开弦构造中允许出现的自洽规范群因子，只能为 $\mathrm{U}(N)$, $\mathrm{SO}(N)$ 和 $\mathrm{USp}(2N)$ ，其中后两类要求弦是未定向的。注意，也可以从开弦两个端点上额外自由度的动力学出发得到这些规范群 [116]。

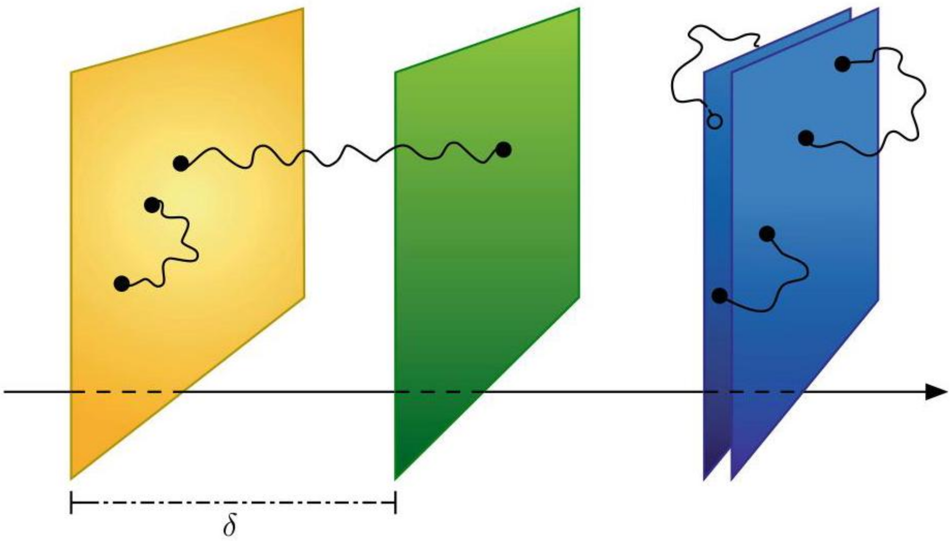

Fig. 2 D-branes and their open strings. The open strings stretching between the yellow and green branes include a massive vector that becomes massless when the relative distance δ goes to zero. The stack of the blue branes yields a non-Abelian gauge group

图 2 D 膜及其开弦。拉伸在黄色膜和绿色膜之间的开弦包含一个大质量矢量，当相对距离 δ 趋于零时该矢量变为无质量。蓝色膜堆给出一个非阿贝尔规范群

Orientifolds

定向轨形

In sections ”Fermionic Strings” and ”Heterotic Strings,” we saw that a useful way to construct new string vacua is to gauge discrete symmetries. This method was employed in building the type 0 theories from type II superstrings, or the non-supersymmetric heterotic $\mathrm{SO}(16)\times\mathrm{SO}(16)$ using the spacetime fermion number, as well as the level-two E_8 theory where the discrete symmetry involves the permutation of the two E_8 factors of the supersymmetric heterotic string. Even the GSO projection itself can be seen as an orbifold construction, where the full spectrum is modded out by the world-sheet fermion number. Following this paradigm, one may wonder if there are additional discrete symmetries whose gauging could lead to new constructions.

在“费米子弦”和“杂化弦”章节中我们已经看到，构造新弦真空的一个常用方法是对离散对称性做规范。该方法曾被用于: 从 II 型超弦出发构建 0 型理论，利用时空费米子数构造非超对称杂化 $\mathrm{SO}(16)\times\mathrm{SO}(16)$ ，以及构造二级 E_8 理论——该理论中的离散对称性涉及超对称杂化弦两个 E_8 因子的置换。甚至 GSO 投影本身也可以看作一种轨形构造，即全谱对世界面费米子数做商。遵循这一范式，我们不禁会问: 是否存在其他额外的离散对称性，对其做规范可以得到新的构造?

To this end, recall that the type IIB theory employs the same GSO projection for left and right movers and thus, although being chiral in spacetime, is non-chiral on the world-sheet. We may then use world-sheet parity $\Omega\ :\ \sigma\to-\sigma$ to restrict its spectrum. This operation, known as the orientifold construction [81,102- 104,106,112,117,118] (for a detailed review with applications to lower-dimensional vacua see [44] and [119]), exchanges the left and right waves on a closed string via the natural action of mapping left-moving oscillators into their right-moving counterparts, i.e., $\alpha_n^\mu\leftrightarrow\tilde{\alpha}_n^\mu$ and similarly for the fermion modes. [2] Under this operation, the graviton and the dilaton, corresponding to the symmetric part of $b^\mu_{-1/2}\tilde{b}^\nu_{-1/2}\,|0\rangle_{\mathrm{L}}\otimes|\,0\rangle_{\mathrm{R}}$, are clearly invariant, while the B -field, corresponding to the anti-symmetric part, is odd and thus it is projected away. A similar projection is also at work in the R-R sector, while only one combination of the fermions in the NS-R and R-NS sectors survives. As usual, a natural way to encode the spectrum of the theory is to compute the partition function, which now involves a projector onto Ω -invariant states,

为此，我们回顾一下:IIB 型弦论对左动模和右动模采用相同的 GSO 投影，因此尽管它在时空上是手征的，在世界面上却是非手征的。我们可以利用世界面 parity $\Omega:\sigma\to-\sigma$ 来限制其能谱。这一操作被称为定向轨形构造 [81,102-104,106,112,117,118](关于其在低维真空的应用的详细综述参见 [44] 和 [119])，它通过将左动振荡器映射为对应右动振荡器的自然作用，交换闭弦上的左行波与右行波，即 $\alpha_n^\mu\leftrightarrow\tilde{\alpha}_n^\mu$ ，费米子模也遵循同样的变换。[2] 在该操作下，对应 $b^\mu_{-1/2}\tilde{b}^\nu_{-1/2}\,|0\rangle_{\mathrm{L}}\otimes|\,0\rangle_{\mathrm{R}}$ 对称部分的引力子和 dilaton 显然是不变的，而对应反对称部分的 B 场是奇的，因此被投影掉。类似的投影也作用在 R-R sector，仅 NS-R 和 R-NS sector 中费米子的一种组合保留下来。和往常一样，计算该理论能谱的自然方式是计算配分函数，此时配分函数包含一个到 Ω 不变态的投影算符，

$$Z=\mathrm{Tr}\left[\frac{1+\Omega}{2}P_{\mathrm{GSO}}q^{L_0}\bar{q}^{\bar{L}_0}\right]\equiv\frac{T+K}{2},\tag{138}$$

where P_{GSO} enforces the GSO projection of type IIB. The first term is nothing but the familiar torus partition function,

其中 P_{GSO} 负责实现 IIB 型的 GSO 投影。第一项就是我们熟悉的环面配分函数，

$$T = \frac{1}{\eta^8(\tau)\,\overline{\eta}^8(\overline{\tau})} |V_8(\tau) - S_8(\tau)|^2 . \tag{139}$$

whereas the second one involves the insertion of the Ω operator in the trace and reads

而第二项包含迹中插入的 Ω 算符，形式为

$$K = \frac{1}{\eta^8(2i\tau_2)} (V_8 - S_8)(2i\tau_2) . \tag{140}$$

Notice that K only receives contributions from the NS-NS and R-R sectors. Taken together with T , it symmetrises the contribution of the NS-NS sector, thus eliminating the B -field from the massless spectrum, and it anti-symmetrises the contribution of the R-R sector, thus eliminating the 0-form and the self-dual 4-form. Ω exchanges the NS-R and R-NS sectors so that only their diagonal combination survives the projection, which is reflected in the 1/2 factor multiplying T . All in all, the massless spectrum enjoys $\mathcal{N} = (1,0)$ supersymmetry and comprises the fields in the supergravity multiplet: the graviton, the dilaton, the R-R 2-form potential, the left-handed gravitino and the right handed dilatino. This chiral spectrum is, however, anomalous and does not define a consistent theory in ten dimensions. An alternative way to see this inconsistency is to understand the topology of the Riemann surface associated to K .

注意 K 仅从 NS-NS 和 R-R 区获得贡献。结合 T ，它对 NS-NS 区的贡献做对称化，从而将 B 场从无质量能谱中剔除；对 R-R 区的贡献做反对称化，从而剔除 0 形式和自对偶 4 形式。Ω 交换 NS-NS 和 R-R 区，因此只有它们的对角组合能在投影后保留，这体现在乘在 T 前的 1/2 因子上。总而言之，无质量能谱具有 $\mathcal{N} = (1,0)$ 超对称，包含超引力多重态中的场: 引力子、dilaton、R-R 2 形式势、左手引力微子和右手 dilatino。然而这个手征能谱是反常的，无法在十维中构成自洽的理论。另一种理解这种不自洽性的方式是分析与 K 关联的黎曼曲面拓扑。

Indeed, the amplitude (140) describes the loop propagation of closed strings with the exchange of left and right movers and, therefore, corresponds to a non-orientable closed Riemann surface of vanishing Euler characteristic, known as the Klein bottle. This can be built from the double-covering torus via the anti-holomorphic involution $z \to 1 - \bar{z} + i\tau_2$, which is compatible with the equivalence relations $z \sim z + n\tau + m$ only if the torus is rectangular with $\tau = 2i\tau_2$. As a result, modular invariance is lost and the Klein bottle admits two alternative descriptions, as depicted in Fig. 3: if the proper time τ_2 is taken to flow vertically, it describes the one-loop propagation of closed strings with an exchange of their left and right waves and the integrated amplitude reads

事实上，振幅 (140) 描述了交换左、右动模式的闭弦的圈传播，因此对应一个欧拉示性数为零的不可定向闭黎曼曲面，即克莱因瓶。它可通过反全纯对合 $z \to 1 - \bar{z} + i\tau_2$ 由双覆盖环面构造而来，该对合仅当环面为矩形且满足 $\tau = 2i\tau_2$ 时才与等价关系 $z \sim z + n\tau + m$ 相容。因此模不变性被破坏，克莱因瓶有两种等价描述，如图 3 所示: 若固有时 τ_2 取为竖直流向，它描述交换左右波的闭弦的单圈传播，积分振幅可写为

$$\mathcal{K} = \int_0^\infty \frac{d\tau_2}{\tau_2^6} \frac{1}{\eta^8(2i\tau_2)} (V_8 - S_8)(2i\tau_2) . \tag{141}$$

[2] Actually, the simplest instance of a closed string invariant under world-sheet parity is the bosonic string itself. However, the resulting theory is not rich enough to reveal the salient features of orientifold constructions and will not be discussed here. We refer the interested reader to the original literature [105, 116, 120, 121] and to [44] for a review.

[2] 实际上，满足世界面宇称不变性的闭弦最简单例子就是玻色弦本身。但该理论不够丰富，无法展现定向轨形构造的核心特征，因此本文不作讨论。感兴趣的读者可参阅原始文献 [105, 116, 120, 121] 以及综述文献 [44]。

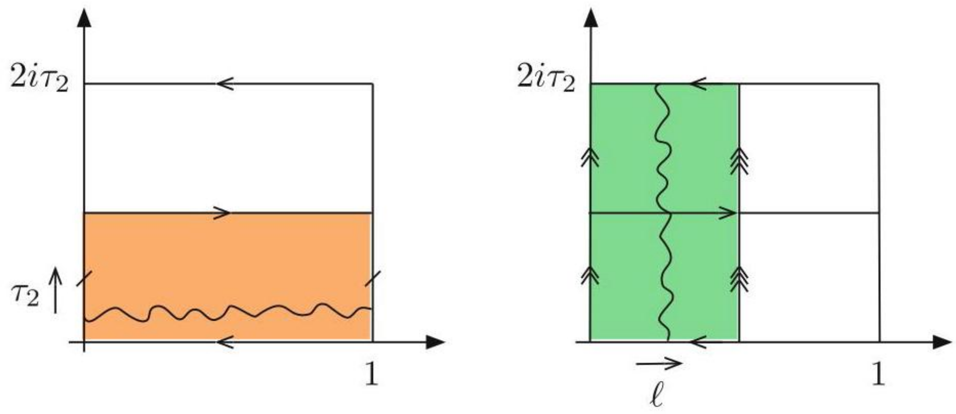

Fig. 3 The left figure shows the one-loop vacuum diagram of a closed string which propagates for a (vertical) proper time τ_2 and flips its orientation. The right figure illustrates the tree-level (horizontal) propagation of a closed string bouncing between the two cross-caps. Both figures also display the double covering torus with modulus $2i\tau_2$

图 3 左图为闭弦的单圈真空图，闭弦在 (竖直) 固有时 τ_2 内传播并翻转其取向。右图阐释闭弦在两个交叉帽之间树级 (水平) 传播。两图也同时给出了带模 $2i\tau_2$ 的双覆盖环面

If the proper time ℓ is instead taken to flow horizontally, which is obtained by the S -modular transformation $\ell = 1/2\tau_2$, the integrated amplitude

若改为固有时 ℓ 沿水平流向，这可通过 S 模变换 $\ell = 1/2\tau_2$ 得到，此时积分振幅

$$\widetilde{\mathcal{K}} = 2^5 \int_0^\infty d\ell \frac{1}{\eta^8(i\ell)} (V_8 - S_8)(i\ell) \tag{142}$$

describes the tree-level propagation of the NS-NS and R-R states between two cross-caps [3] [103-106], also known as orientifold planes (O-planes, for short), see Fig. 3. In the limit of an infinitely long tube, $\ell \to \infty$, only massless excitations survive and the resulting diagram involves 1-point functions coupling the massless fields to the cross-caps, and an on-shell propagator evaluated at zero momentum. This shows that also orientifold planes carry tension as well as charge for the R-R potentials. Since the Ω involution preserves the ten-dimensional Lorentz symmetry, the O-planes invade the whole space-time and, therefore, can only couple to the R-R 10-form potential C_{10} . This field is (trivially) non-dynamical and its equation of motion implies that the net charge of its sources must vanish, which is not the case if only orientifold planes are present. This problem can be solved by the introduction of D9 branes, which are also charged with respect to C_{10} and may yield a neutral configuration.

描述 NS-NS 态和 R-R 态在两个交叉帽 [3] 之间的树级传播 [103-106]，交叉帽也称为定向轨形平面 (简称 O 平面)，见图 3。在传播管无限长的极限 $\ell \to \infty$ 下，仅无质量激发留存，该图包含无质量场与交叉帽耦合的 1 点函数，以及零动量下的 on-shell 传播子。这说明 O 平面也携带张力和 R-R 势的荷。由于 Ω 对合保持十维洛伦兹对称性，O 平面覆盖整个时空，因此只能耦合 R-R 10-形式势 C_{10}。该场 (平凡地) 非动力学，其运动方程要求源的总荷为零，仅存在 O 平面时无法满足这一条件。引入 D9 膜即可解决该问题，D9 膜也携带 C_{10} 对应的荷，可形成中性组态。

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[3] A cross-cap is the simplest unoriented Riemann surface with Euler characteristic $\chi = 1$ and can be built from the double-covering sphere by identifying antipodal points, as shown in Fig. 4. The resulting surface has no boundaries and is unoriented.

[3] 交叉帽是欧拉示性数为 $\chi = 1$ 的最简单不可定向黎曼曲面，可通过对双覆盖球面对跖点认同构造而来，如图 4 所示。最终得到的曲面没有边界，且不可定向。

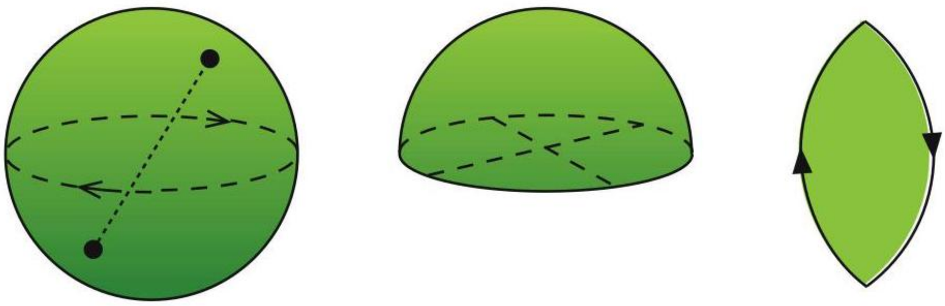

Fig. 4 The cross-cap is obtained by identifying antipodal points on the sphere. The sides of the fundamental domain are identified following the direction of the arrows, so that the surface has no boundaries and is unoriented

图 4 交叉帽由球面认同对跖点得到。基本域的边按箭头方向认同，因此最终曲面没有边界，且不可定向

Since the closed strings considered in this section are unoriented, so should be the open strings living on the D9 branes. For unoriented open strings, the two endpoints are equivalent and, hence, the worldsheet parity exchanges them and acts as $\Omega : \sigma \to 1 - \sigma$. This action translates to

由于本节讨论的闭弦是不可定向的，因此 D9 膜上的开弦也应为不可定向。对于不可定向开弦，两个端点等价，因此世界面宇称会交换两个端点，其作用为 $\Omega : \sigma \to 1 - \sigma$。该作用对应

$$\alpha_n \to (-1)^n \alpha_n, \ d_n \to (-1)^n d_n, \ b_r \to (-1)^{r-1/2} b_r. \tag{143}$$

As a result, the spectrum of unoriented open strings is encoded into

因此，不可定向开弦的谱可编码为

$$\mathrm{Tr}\left[\frac{1+\Omega}{2}P_{\mathrm{GSO}}q^{L_0}\right]. \tag{144}$$

The trace involving the identity corresponds to the annulus amplitude computed in section "Open Strings and D-Branes" and reads

含单位元的迹对应“开弦与 D 膜”一节计算的环带振幅，其表达式为

$$\mathcal{A} = N^2\int_0^\infty \frac{d\tau_2}{\tau_2^6}\frac{1}{\eta^8\left(\frac{i\tau_2}{2}\right)}(V_8 - S_8)\left(\frac{i\tau_2}{2}\right), \tag{145}$$

where we have introduced a stack of N D9 branes. The multiplicity factor N^2 is due to the fact that open strings can start and end on any of the D-branes in the stack and reflects the possibility of introducing Chan-Paton charges at the string endpoints [115, 116, 122] . The trace involving the world-sheet parity Ω reads

其中我们引入了一叠 N 张 D9 膜。多重因子 N^2 源于开弦可以从叠中任意 D 膜出发并终止于叠中任意 D 膜这一事实，反映了弦端点 [115, 116, 122] 可以引入陈-帕顿电荷的性质。涉及世界面宇称 Ω 的迹写为

$$\mathcal{M} = \varepsilon N\int_0^\infty \frac{d\tau_2}{\tau_2^6}\frac{1}{\hat{\eta}^8\left(\frac{1}{2}+\frac{i\tau_2}{2}\right)}\left(\hat{V}_8 - \hat{S}_8\right)\left(\frac{1}{2}+\frac{i\tau_2}{2}\right). \tag{146}$$

Here, the 1/2 shift in the argument reflects the action (143) of Ω on the oscillators, the hatted characters defined as in (133) remove the fictitious overall phase, while the sign ε is ascribed to the parity ambiguity of the vacuum and will be determined shortly. In this case, the multiplicity scales like N since the open strings must start and end on the same D-brane so that the system respects the world-sheet parity that exchanges the two endpoints.

此处，宗量的 1/2 平移反映了 Ω 对振荡模式的作用 (143)，按 (133) 定义的带帽量消除了人为引入的整体相位，而符号 ε 归因于真空的宇称不确定性，我们很快就会确定它。在这种情况下，多重度按 N 标度，这是因为开弦必须起止在同一张 D 膜上，才能让系统满足交换两个端点的世界面宇称。

Also this amplitude has a geometrical interpretation in terms of a Riemann surface spanned by an open string which exchanges its endpoints. This unoriented Riemann surface has a boundary and it is known as the Möbius strip, see Fig. 5. As in the previous cases, it can be built via an anti-holomorphic involution of a double-covering torus whose modulus now has a fixed real part $\tau = \frac{1}{2} + \frac{i\tau_2}{2}$. Also here, there is no notion of modular invariance and, in fact, also the Möbius strip admits two alternative descriptions depending on whether the proper time is taken to flow vertically or horizontally. In the former case, it describes the one-loop propagation of open strings which exchange their end-points while, in the latter, closed strings freely propagate between a boundary and a cross-cap [103-106], as shown in Fig. 5. In the limit of an infinitely long tube, only massless states propagate and, in the NS-NS sector, and the resulting diagram involves the product of the tensions D9 branes and O9 planes, times the on-shell propagator at zero momentum. Similarly, in the R-R sector only the non-dynamical 10-form potential probes both the D9 branes and the O9 planes and, therefore, the amplitude is proportional to the product of the two charges. The sign ambiguity ε reflects the possibility that the tensions and charges of D-branes and O-planes have the same or opposite sign.

该振幅同样可以通过交换端点的开弦张成的黎曼曲面给出几何诠释。这个无定向黎曼曲面带有一个边界，被称为莫比乌斯带，见图 5。和之前的情况一样，它可以通过二重覆盖环面的反全纯对合构造得到，该环面的模现在具有固定实部 $\tau = \frac{1}{2} + \frac{i\tau_2}{2}$ 。此处同样不存在模不变性的概念，事实上根据固有时间取为垂直流向还是水平流向，莫比乌斯带也有两种不同的描述。前者描述交换端点的开弦的单圈传播，后者描述闭弦在边界和叉帽之间的自由传播 [103-106]，如图 5 所示。在无限长管道极限下，只有无质量态传播，在 NS-NS sector 中，最终的图由 D9 膜和 O9 平面的张力乘积，乘以零动量下的在壳传播子给出。类似地，在 R-R sector 中只有非动力学 10 形式势可以探测 D9 膜和 O9 平面，因此振幅正比于二者电荷的乘积。符号不确定性 ε 反映了 D 膜和 O 平面的张力与电荷可以同号也可以异号。

As usual, an S -modular transformation brings the direct (loop) channel representation (145) into the transverse (tree-level) representation

和通常情况一样，一次 S 模变换将直 (圈) 通道表示 (145) 变为横 (树图级) 通道表示

$$\widetilde{\mathcal{A}} = 2^{-5}N^2\int_0^\infty d\ell \frac{1}{\eta^8(i\ell)}(V_8 - S_8)(i\ell)\,. \tag{147}$$

In the case of the Möbius amplitude, this map is realised by the transformation

对于莫比乌斯振幅，该映射由如下变换实现

$$P:\ \frac{1}{2} + \frac{i\tau_2}{2} \to \frac{1}{2} + \frac{i}{2\tau_2}, \tag{148}$$

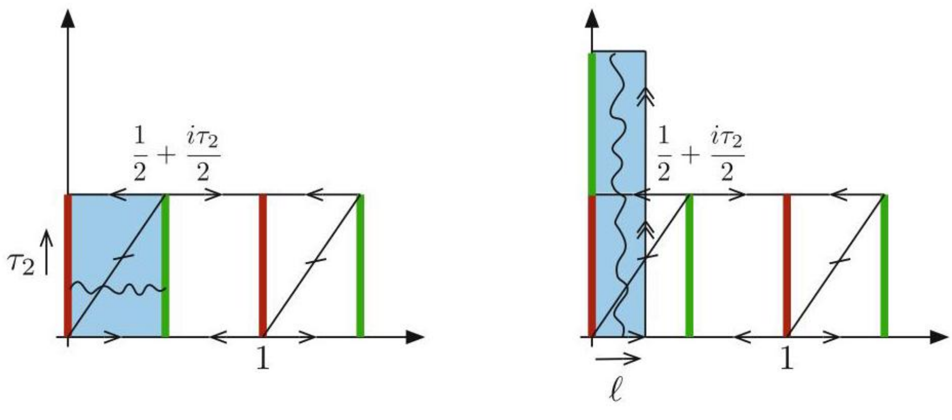

Fig. 5 The left figure shows the one-loop vacuum diagram of an open string which propagates for a (vertical) proper time τ_2 and flips its orientation; the end-points trace the single boundary of the Möbius strip composed by the red and green vertical lines. The right figure shows the tree-level (horizontal) propagation of a closed string bouncing between a boundary and a cross-cap. Both figures also display the double covering torus with modulus $\frac{1}{2} + \frac{i\tau_2}{2}$

图 5 左图展示了改变取向的开弦在 (垂直) 固有时间 τ_2 内传播的单圈真空图；端点勾勒出由红色和绿色竖线构成的莫比乌斯带的单一边界。右图展示了闭弦在边界和叉帽之间反弹的树图级 (水平) 传播。两张图也都展示了模为 $\frac{1}{2}+\frac{i\tau_2}{2}$ 的二重覆盖环面

with ${}^4P = TST^2S$ [81], and S, T being the standard generators of $\mathrm{SL}(2;\mathbb{Z})$. The resulting transverse channel Möbius amplitude, thus, reads

其中 ${}^4P = TST^2S$ [81]，且 S, T 是 $\mathrm{SL}(2;\mathbb{Z})$ 的标准生成元。因此横通道下得到的莫比乌斯振幅写为

$$\widetilde{\mathcal{M}} = 2\varepsilon N \int_0^\infty d\ell \frac{1}{\hat{\eta}^8\left(\frac{1}{2}+i\ell\right)} \left(\hat{V}_8 - \hat{S}_8\right)\left(\frac{1}{2}+i\ell\right). \tag{149}$$

In the $\ell \to \infty$ limit, aside from a multiplicative overall divergence ascribed to the massless propagator at zero momentum, $\widetilde{\mathcal{K}} + \widetilde{\mathcal{A}} + \widetilde{\mathcal{M}}$ yield the perfect square

在 $\ell \to \infty$ 极限下，除了零动量无质量传播子带来的整体发散乘因子外，$\widetilde{\mathcal{K}} + \widetilde{\mathcal{A}} + \widetilde{\mathcal{M}}$ 给出完全平方

$$2^5 + 2^{-5}N^2 + 2\varepsilon N = 2^{-5}\left(N + 2^5\varepsilon\right)^2. \tag{150}$$

as schematically depicted in Fig. 6. The term inside the parenthesis on the r.h.s. is nothing but the overall tension of D-branes and O-planes in the NS-NS sector, while, it equals the overall 10-form charge in the R-R sector. The consistency of the C_{10} equations of motion requires the tadpole cancellation condition

如图 6 的示意描绘。右侧括号内的项正是 NS-NS sector 中 D 膜和 O 平面的整体张力，同时它等于 R-R sector 中的整体 10 形式电荷。C_{10} 运动方程的自洽性要求蝌蚪图消除条件为

$$N + 2^5\varepsilon = 0, \tag{151}$$

which guarantees a neutral configuration in accordance with Gauss' law. The unique solution is $\varepsilon = -1$ and $N = 32$. In this supersymmetric setup, the cancellation of R-R tadpoles also guarantees the cancellation of NS-NS ones, since they are related by supersymmetry transformations. Assuming that the physical D-branes have positive tension and charge, this implies that the orientifold planes involved in this construction have negative tension and charge and are known in the literature as O9_planes.

这保证了符合高斯定律的中性构型。唯一解为 $\varepsilon = -1$ 和 $N = 32$ 。在该超对称构型中, R-R tadpole(拉蒙德-拉蒙德 tadpole) 的抵消也保证了 NS-NS(内沃-施瓦茨-内沃-施瓦茨)tadpole 的抵消，因为二者通过超对称变换关联。假设物理 D 膜具有正张力和正电荷，这意味着该构造中涉及的 O 定向面具有负张力和负电荷，在文献中被称为 O9 平面。

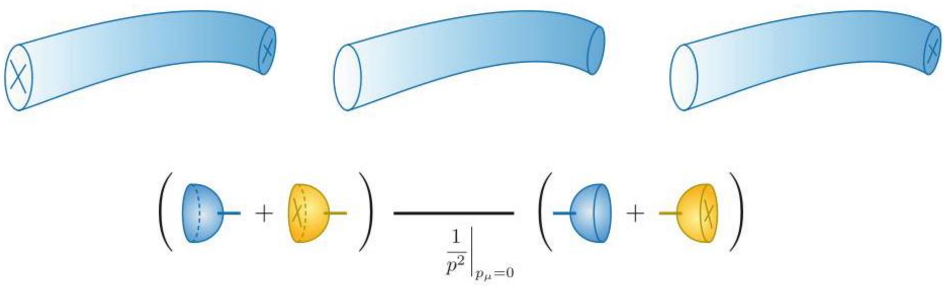

Fig. 6 In the $\ell \to \infty$ limit, the transverse-channel Klein bottle, annulus and Möbius strip amplitudes probe the tension and the charge of O-planes and D-branes. The contributions $\widetilde{\mathcal{K}}+\widetilde{\mathcal{A}}+\widetilde{\mathcal{M}}$ factorise into the square of the tadpoles times the zero-momentum propagator of the corresponding massless state

图 6 在 $\ell \to \infty$ 极限下，横道克莱因瓶振幅、环带振幅和默比乌斯带振幅反映 O 平面和 D 膜的张力与电荷。贡献项 $\widetilde{\mathcal{K}}+\widetilde{\mathcal{A}}+\widetilde{\mathcal{M}}$ 分解为 tadpole 的平方乘以对应无质量态的零动量传播子

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[4] Actually, on the hatted characters this transformation acts as $P = T^{1/2}ST^2ST^{1/2}$.

[4] 实际上，该变换对带帽字符的作用为 $P = T^{1/2}ST^2ST^{1/2}$ 。

Returning to the direct channel annulus and Möbius strip amplitudes, the tadpole condition uniquely fixes the spectrum which, at the massless level, comprises an $\mathcal{N}=(1,0)$ vector multiplet in the adjoint representation of SO(32). Together with the (unoriented) closed string spectrum this yields the field content of type I superstrings, which is free of irreducible gravitational and gauge anomalies, as guaranteed by the cancellation of R-R tadpoles [123-126]. Although the type I superstring shares the same massless spectrum as the SO(32) heterotic string, the elementary degrees of freedom are obviously completely different. Because the various couplings in the low energy effective actions emerge from different Riemann surfaces, the map between the two is non-perturbative since it inverts the string coupling constants, $g_{s,\,\text{het}} = 1/g_{s,\text{I}}$ [108 − 110, 127] .

回到直道环带振幅和默比乌斯带振幅，tadpole 条件唯一确定了能谱: 在无质量能级，该能谱包含一个属于 SO(32) 伴随表示的 $\mathcal{N}=(1,0)$ 矢量多重态。结合 (非定向) 闭弦能谱，我们就得到了 I 型超弦的场内容，正如 R-R tadpole 抵消所保证的，该理论不存在不可约引力反常和规范反常 [123-126]。尽管 I 型超弦与 SO(32) 杂化弦具有相同的无质量能谱，二者的基本自由度显然完全不同。由于低能有效作用中的各类耦合来自不同的黎曼曲面，两种理论之间的映射是非微扰的——因为映射会反转弦耦合常数，即 $g_{s,\,\text{het}} = 1/g_{s,\text{I}}$ [108 − 110, 127] 。

Actually, this is not the only vacuum that one may construct as an orientifold of the type IIB theory. In fact, although R-R tadpoles must always be cancelled for the theory to be unitary, NS-NS ones are not associated to any conservation law and, therefore, can be relaxed at the cost of modifying the background [128, 129]. This new vacuum has the same Klein bottle (141) and annulus (145) amplitudes, but the Möbius amplitudes now read

事实上，这并不是我们可以从 IIB 型理论的定向形构造得到的唯一真空。实际上，尽管为了保证理论的幺正性，R-R tadpole 必须始终被抵消，NS-NS tadpole 并不与任何守恒定律关联，因此可以通过修改背景为代价放宽该要求 [128, 129]。这个新真空具有和原真空相同的克莱因瓶振幅 (141) 和环带振幅 (145)，但默比乌斯振幅现在变为:

$$\mathcal{M} = N\int_0^\infty \frac{d\tau_2}{\tau_2^6}\frac{1}{\hat{\eta}^8\left(\frac{1}{2}+\frac{i\tau_2}{2}\right)}\left(\hat{V}_8+\hat{S}_8\right)\left(\frac{1}{2}+\frac{i\tau_2}{2}\right), \tag{152}$$

and

和

$$\widetilde{\mathcal{M}} = 2N\int_0^\infty d\ell\frac{1}{\hat{\eta}^8\left(\frac{1}{2}+i\ell\right)}\left(\hat{V}_8+\hat{S}_8\right)\left(\frac{1}{2}+i\ell\right). \tag{153}$$

From the latter, we can extract that the tensions of O-planes and D-branes have the same sign, while their R-R charges have opposite signs. Indeed, this vacuum involves what is known as O_{9+} planes (with positive tension and R-R charge) and anti-D9 branes (with positive tension and negative R-R charge). The R-R tadpole cancellation conditions still select $N = 32$, while the NS-NS tadpoles are not cancelled and induce the potential

从中我们可以得出，O 平面与 D 膜的张力符号相同，而它们的 R-R 电荷符号相反。实际上，该真空涉及的就是所谓的 O_{9+} 平面 (具有正张力和正 R-R 电荷) 和反 D9 膜 (具有正张力和负 R-R 电荷)。R-R tadpole 抵消条件仍选中 $N = 32$ ，而 NS-NS tadpole 未被抵消，并诱导出势能:

$$\int d^{10}x\sqrt{-G}V(\Phi) \sim (N+32)\int d^{10}x\sqrt{-G}e^{-\Phi}, \tag{154}$$

in the string frame. The dependence on the dilaton originates from the coupling to the disk and the cross-caps, which both have Euler characteristic $\chi = 1$. This vacuum is known as the Sugimoto model [130]. It is characterised by a supersymmetric closed string spectrum containing the massless $\mathcal{N} = (1,0)$ supergravity multiplet, while supersymmetry is explicitly broken in the open string sector, whose massless excitations now include a vector in the adjoint representation of USp(32) and a left-handed fermion in the 496 anti-symmetric representation.

在弦坐标系中。对 dilaton(伸缩子) 的依赖来自它对圆盘和交叉帽的耦合，二者的欧拉示性数均为 $\chi = 1$ 。该真空被称为杉本模型 [130]。它的特征是具有包含无质量 $\mathcal{N} = (1,0)$ 超引力多重态的超对称闭弦能谱，而开弦部分的超对称被显式破缺: 开弦的无质量激发现在包含一个属于 USp(32) 伴随表示的矢量，以及一个属于 496 维反对称表示的左手征费米子。

Also in this case the cancellation of the R-R tadpoles fix the gauge group and the representation of the fermions and ensures the cancellation of irreducible anomalies [123-126]. Notice that, in this case, the anti-symmetric representation 496 is reducible and decomposes as $\mathbf{496} = \mathbf{495} + \mathbf{1}$. The singlet fermion plays the role of the Goldstino and, together with the dilaton potential (154), implies that in the open string sector supersymmetry is still present, albeit non-linearly realised [131, 132].

在该情形下, R-R tadpole 抵消同样固定了规范群和费米子的表示, 并保证了不可约反常的抵消 [123-126]。请注意，在该情形下，496 维反对称表示是可约的，可以分解为 $\mathbf{496} = \mathbf{495} + \mathbf{1}$ 。单态费米子充当戈德斯蒂诺，结合 (154) 的伸缩子势，这意味着开弦部分的超对称仍然存在，只是以非线性方式实现 [131, 132]。

This vacuum is free of tachyonic instabilities. However, the emergence of the dilaton potential, and thus a non-trivial interaction between O_+ -planes and anti-branes, implies a back-reaction on the Minkowski vacuum breaking the $SO(1,9)$ down to $SO(1,8)$ [133].

该真空无快子不稳定性。然而，膨胀子势的出现，以及由此产生的 O_+ 平面和反膜之间的非平凡相互作用，意味着对闵氏真空的反作用会将 $SO(1,9)$ 破缺为 $SO(1,8)$ [133]。

The type IIB superstring is not the only closed string theory which is left-right symmetric. In fact, although heterotic strings are asymmetric by construction and type IIA involves different GSO projections for the left and right movers, the type 0 theories are Ω -invariant and admit an orientifold projection. In the following, we shall focus on type 0B orientifolds, and refer the interested reader to the original work [81] and to [44] for a discussion of the 0 A case.

IIB 型超弦并非唯一的左右对称闭弦理论。实际上，尽管杂化弦在构造上是不对称的，且 IIA 型对左行和右行模式采用不同的 GSO 投影，但 0 型理论是 Ω 不变的，允许定向形投影。在下文中，我们将聚焦于 0B 型定向形，感兴趣的读者可参阅原始文献 [81] 和 [44] 了解 0 A 的情况。

The type 0B theory (118) involves the two R-R sectors $|S_8|^2$ and $|C_8|^2$ and, therefore, two different (non-dynamical) 10-form potentials are present. As a result, the O9 planes and D9 branes of the OB theory are charged with respect to both of them, in addition to carrying a non-trivial tension. This allows for a richer pattern of O-planes and D-branes with vanishing R-R charges.

0B 型理论 (118) 包含两个 R-R 扇区 $|S_8|^2$ 和 $|C_8|^2$ ，因此存在两个不同的 (非动力学)10 形式势。由此，0B 理论的 O9 平面和 D9 膜除了具有非平凡张力外，还对这两个势带有电荷。这为 RR 电荷为零的 O 平面和 D 膜提供了更丰富的构型模式。

The standard orientifold projection of $\mathcal{T}_{0\,\mathrm{B}}$ starts with the Klein bottle amplitude [81]

$\mathcal{T}_{0\,\mathrm{B}}$ 的标准定向形投影从克莱因瓶振幅开始 [81]

$$\mathcal{K} = O_8 + V_8 - S_8 - C_8, \tag{155}$$

where, to lighten the notation, we henceforth do not explicitly display the integral nor the contribution of the world-sheet bosons, since they are unambiguous and may easily be reconstructed. We will, however, take them into account when deriving the transverse channel amplitudes. The projected closed string spectrum in $(\mathcal{T}_{0\,\mathrm{B}} + \mathcal{K})/2$ comprises the metric, the dilaton, two R-R 2-forms, as well as a tachyon, which clearly makes the system unstable. This orientifold is consistent and does not require the introduction of D-branes. In fact, the transverse channel Klein bottle reads

为简化记号，此后我们不再显式写出积分，也不单独标出世界面玻色子的贡献，因为它们是明确的，很容易复原。不过我们在推导横向道振幅时会将其考虑在内。$(\mathcal{T}_{0\,\mathrm{B}}+\mathcal{K})/2$ 中投影后的闭弦谱包含度规、膨胀子、两个 R-R 2 形式以及一个快子，这显然会导致系统不稳定。该定向形是自洽的，不需要引入 D 膜。实际上，横向道克莱因瓶可写为

$$\widetilde{\mathcal{K}} = 2^6 V_8 \tag{156}$$

and only NS-NS states propagate in the tube between the two cross-caps. As a result, the configuration of O-planes present in this construction has an overall tension but vanishing R-R charges. Hence, Gauss' law for both C_{10} ’ s is no longer violated, and the construction is self-consistent, although the dilaton tadpole in (156) induces a potential of the form (154), as in the Sugimoto model. Open strings can be added at the cost of introducing additional tachyons. In fact, the configuration of the D-branes involved must be neutral with respect to the two C_{10} forms present in the

且只有 NS-NS 态在两个交叉帽之间的管中传播。因此，该构造中 O 平面的构型总张力非零但 R-R 电荷为零。由此，两个 C_{10} 都不再违反高斯定理，构造是自洽的，只不过 (156) 中的膨胀子蝌蚪会诱导出形式为 (154) 的势，这和杉本模型的情况一致。增加开弦就需要引入额外的快子。实际上，涉及的 D 膜构型必须对 0B 谱中存在的两个 C_{10} 形式保持中性，

0B spectrum, and this can only happen if brane-anti-brane pairs are present, which inevitably contain a tachyon stretching between a brane and an anti-brane. This open string sector was built in [81], and we refer the reader to the original work for further details and to [44] for a description in terms of D-branes.

而这只有存在膜-反膜对时才能实现，膜和反膜之间必然延伸出一个快子。这种开弦扇区已在 [81] 中构造完成，更多细节请读者参阅原始文献，关于 D 膜的描述可参阅 [44]。

Actually, for type 0 B orientifolds the action of Ω on the closed string sector is not unique and, in fact, two more choices are possible [134,135]. This is an instance of a more general framework whereby different choices of Klein bottle projections are allowed whenever simple currents are present in the CFT [136-142]. In the first case, Ω symmetrises states both in the NS-NS and R-R sectors and is associated to the Klein bottle amplitude

事实上，对于 0 B 型定向形，Ω 在闭弦扇区的作用不是唯一的，实际上还有另外两种可能的选择 [134, 135]。这是一个更一般框架下的例子: 只要共形场论中存在单纯流，就允许选择不同的克莱因瓶投影 [136-142]。第一种情况中，Ω 对 NS-NS 扇区和 R-R 扇区的态都做对称化，对应克莱因瓶振幅为

$$\mathcal{K}' = O_8 + V_8 + S_8 + C_8. \tag{157}$$

The unoriented closed-string spectrum now comprises a tachyon, the graviton, the dilaton, two R-R scalars and a 4-form potential. Also in this case, the vacuum is consistent and does not require the introduction of open strings, since

此时未定向化的闭弦谱包含一个快子、引力子、膨胀子、两个 R-R 标量和一个 4 形式势。这种情况下真空同样是自洽的，不需要引入开弦，因为

$$\widetilde{\mathcal{K}}' = 2^6 O_8 \tag{158}$$

indicates once more that the configuration of O-planes has vanishing R-R charges, which guarantees compatibility with Gauss' law for the C_{10} potentials. Nevertheless, as in the previous case, open strings can be added by introducing pairs of branes and anti-branes, which induce further tachyonic instabilities, as discussed in [134, 135].

再次表明 O 平面构型的 R-R 电荷为零，这保证了与 C_{10} 势的高斯定理相容。尽管如此，和前一种情况一样，增加开弦需要引入膜-反膜对，这会引发额外的快子不稳定性，相关讨论见 [134, 135]。

In the second case, the Klein bottle instead reads [5]

第二种情况中，克莱因瓶则为 [5]

$$\mathcal{K}'' = -O_8 + V_8 + S_8 - C_8, \tag{159}$$

and has the virtue of projecting away the tachyon, since it is now odd under Ω . Therefore, the closed string sector is non-tachyonic and its massless spectrum includes the graviton and the dilaton from the NS-NS sector, as well as a scalar, a 2- form and a self-dual 4-form from the R-R sectors. This chiral spectrum is anomalous as reflected in the non-trivial R-R tadpole in

其优点是投影消除了快子，因为快子在 Ω 作用下是奇的。因此，闭弦扇区没有快子，其无质量谱包含 NS-NS 扇区的引力子和胀子，以及 R-R 扇区的一个标量、一个 2-形式和一个自对偶 4-形式。这种手征谱是反常的，体现为非平凡的 R-R 蝌蚪存在于

$$\mathcal{K}'' = -2^6 C_8 \tag{160}$$

To cancel it, open strings must now be added and involve two types of D9 branes with charges $(+,+)$ and $(-,+)$ with respect to the two 10 -forms originating from $|S_8|^2$ and $|C_8|^2$, respectively. This is reflected in the coefficients of the S_8 and C_8 characters in the transverse channel annulus amplitude

为抵消该蝌蚪，必须引入开弦，涉及两类 D9 膜，它们分别对起源于 $|S_8|^2$ 和 $|C_8|^2$ 的两个 10-形式带电荷 $(+,+)$ 和 $(-,+)$ 。这体现在横通道环面振幅中 S_8 和 C_8 特征标系数上

$$\widetilde{\mathcal{A}}'' = 2^{-6}\left[\left(N+\bar{N}\right)^2(V_8-C_8)-\left(N-\bar{N}\right)^2(O_8-S_8)\right]. \tag{161}$$

[5] Clearly, there is an equivalent option, whereby the S_8 and C_8 characters are interchanged.

[5] 显然，存在一个等价选择，即交换 S_8 和 C_8 特征标。

Together with

结合

$$\widetilde{\mathcal{M}}'' = 2\left(N + \bar{N}\right)\hat{C}_8, \tag{162}$$

$\widetilde{\mathcal{K}}''$ and $\widetilde{\mathcal{A}}''$ yield the tadpole conditions

$\widetilde{\mathcal{K}}''$ 和 $\widetilde{\mathcal{A}}''$ 给出蝌蚪条件

$$N + \bar{N} = 64,\ N - \bar{N} = 0. \tag{163}$$

The direct channel annulus and Möbius strip amplitudes read

直通道环面和莫比乌斯带振幅可写为

$$\mathcal{A}'' = 2N\bar{N}V_8 - \left(N^2 + \bar{N}^2\right)C_8, \tag{164}$$

and

和

$$\mathcal{M}'' = \left(N + \bar{N}\right)\hat{C}_8. \tag{165}$$

The open string spectrum is also non-tachyonic and contains gauge bosons in the adjoint representation of U(32) and right-handed fermions in the anti-symmetric $\mathbf{496} \oplus \overline{\mathbf{496}}$ representations, precisely as needed to cancel the contribution of the self-dual 4-form to the gravitational anomaly. Being in the anti-symmetric representations of U(32), the theory is also free from gauge anomalies, although R-R forms of different degree are at work to cancel the non-Abelian and Abelian parts, thus generalising the Green-Schwarz mechanisms which is normally at work in ten [92] and six [143] dimensions. This is the celebrated type 0′B vacuum of Sagnotti [134, 135]. Together with the SO (16) × SO (16) heterotic string and the Sugimoto model, they are the only non-supersymmetric, non-tachyonic vacua in ten dimensions.

开弦谱同样没有快子，包含 U(32) 伴随表示的规范玻色子，以及反对称 $\mathbf{496} \oplus \overline{\mathbf{496}}$ 表示的右手费米子，这正好满足抵消自对偶 4-形式对引力反常贡献的要求。由于属于 U(32) 的反对称表示，该理论也不存在规范反常，不同次数的 R-R 形式分别抵消了非阿贝尔和阿贝尔部分的反常，推广了通常十维 [92] 和六维 [143] 中生效的格林-施瓦茨机制。这就是著名的 Sagnotti 的 0′B 真空 [134, 135]。它与 SO (16) × SO (16) 杂化弦、杉本模型一起，是十维中仅有的三个非超对称、无快子的真空。

Orientifold vacua in lower dimensions exhibit a much richer structure involving O-planes and D-branes of different dimensionality and various patterns of (partial) supersymmetry breaking. However, they will not be considered here and we refer the reader to [44] and [119] for reviews.

低维的定向模真空具有更丰富的结构，包含不同维数的 O-平面和 D 膜，以及多种 (部分) 超对称破缺模式。本文不对此展开讨论，读者可参阅综述文献 [44] 和 [119]。

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